Electromagnetic Radiation and the Coming of Age of the Equivalence Principle

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The discussion here ranges in a recycling and sometimes redundant way over the following:

- Uniformly accelerating charged particles.
- Supported charges in static homogeneous gravitational fields.
- Electromagnetic radiation from such charges.
- Equivalence principles.

The aim is to give an overview of the book S.N. Lyle: *Uniformly Accelerating Charged Particles. A Threat to the Equivalence Principle*, Springer, Heidelberg (2008), and to raise a few questions about this long-standing and sometimes heated controversy.

Consider first a charged particle coming down the x axis in a flat spacetime, slowing to a halt somewhere, then accelerating back up the x axis in such a way that its four-acceleration has constant relativistic length $a^\mu a_\mu$. This is eternal uniform acceleration, illustrated by the worldline in Fig. 1. Uniform acceleration means that the worldline is a hyperbola, while eternal means that it goes on forever and has been going on forever.

Now accelerating charged particles usually radiate electromagnetic energy, so what about this point charge with hyperbolic motion? In order to find out, we have to solve Maxwell's equations for the fields, and luckily we always have the Lienard–Wiechert retarded solutions. In this case we find that fields are produced in the region $x + t \geq 0$, and we notice something interesting at the instant of time $t = 0$, when the charge is instantaneously at rest in this particular inertial frame, namely that the magnetic fields are instantaneously equal to zero everywhere, but only for this instant of time. The same can therefore be said of the Poynting vector.

Of course, a Lorentz transformation can reduce any point on the worldline to rest, and as Pauli pointed out, the hyperbolic worldline looks exactly the same in any inertial frame, so the magnetic field and hence the Poynting vector are always zero everywhere, provided we keep changing inertial frame, so that we

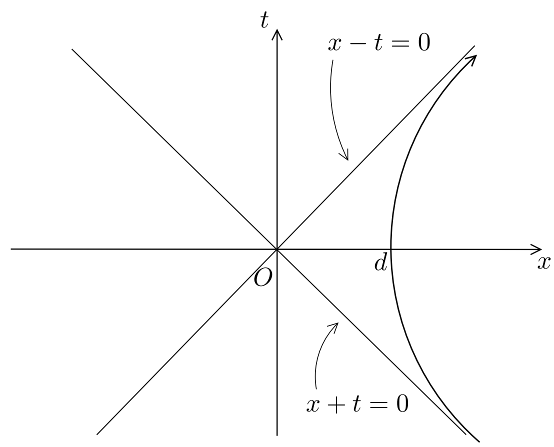

Figure 1: A charged particle arrives from large positive x (*bottom right*), slows down to a halt at $x = d$ (in this frame), then accelerates back up the x axis. The worldline is asymptotic to the null cones at the origin, i.e., it is asymptotic to $x + t = 0$ for large negative times, and $x - t = 0$ for large positive times. Naturally, it never actually reaches the speed of light

are always using the inertial frame instantaneously comoving with the charge. For this reason, Pauli suggested that the charge might not radiate [1].

But in any given inertial frame, the Poynting vector is going to change from zero as time goes by, and as far as we know, there is no relevance in what one would observe by continually changing inertial frame. However, in a well known non-inertial frame adapted to the motion of the charge, the magnetic field components are identically zero everywhere and at all times. In fact, for any timelike worldline in Minkowski spacetime, there are coordinates with special properties, said to be adapted to the worldline, and sometimes called semi-Euclidean (SE) coordinates for reasons to become clear in a moment.

The idea is that, at each event on the given worldline, the accelerating observer borrows the hyperplane of simultaneity of an instantaneously comoving inertial observer and attributes her own proper time to all points on it. With this ploy and a few other simple tricks [2, Chap. 2], we can arrange for the observer to sit permanently at the space origin of the new coordinate system, whence her worldline is just the time coordinate axis, with the time coordinate being the observer's own proper time. We can also arrange for the metric to have Minkowski form right along the time axis, but not off it. This means that the coordinate frame we are constructing is a tetrad frame along the worldline, but not off it. And finally, by the ploy mentioned above, the geometry will be Euclidean on the constant time hypersurfaces, hence the name semi-Euclidean coordinates. Note that the connection is *not* zero along the worldline, since it must encode the acceleration.

We can make another interesting observation here, which shows just how special this kind of motion is. It is only for an eternally uniformly accelerating observer that the Minkowski metric still has static form when expressed relative to SE coordinates. For any other kind of acceleration, if we carry out this kind of construction, the components of the Minkowski metric will depend on the new time coordinate. (For a proof in the case of acceleration along a straight line, see [2, Chap. 2]. For a general proof, see [3, Chap. 12].)

We now have the non-inertial coordinate system adapted to the charge motion, but we still need to be able to talk about magnetic fields in a situation where we are not using inertial coordinates. Physically, this is not so obvious, but mathematically it is very easy, because we have the electromagnetic field tensor $F_{\mu\nu}$ and we can express its components relative to any frame. We also know that the matrix of components of this tensor is antisymmetric in any frame so it can always be written in the form

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & -B_3 & B_2 \\ -E_2 & B_3 & 0 & -B_1 \\ -E_3 & -B_2 & B_1 & 0 \end{pmatrix} , \tag{1}$$

and then we can just read off $\boldsymbol{E}$ and $\boldsymbol{B}$. What these mean physically for a general coordinate frame is another matter, to which we shall return.

So if we take the charge with hyperbolic motion, find its Lienard–Wiechert retarded fields relative to some inertial coordinate system, then transform them to the SE coordinate system adapted to the charge motion, we find that the SE magnetic field is permanently zero. In addition, the SE electric field is static, in the sense that, at any given SE space coordinate, the SE electric field does not change as SE coordinate time goes by. We may make another observation here that shows once again just how special this kind of motion is: it is *precisely and only* for the case of eternal uniform acceleration that this construction yields such an elegant and simple picture.

So maybe Pauli was right after all. Maybe there is no radiation of electromagnetic energy in this case. There is another reason for thinking that this may be so. If a charge radiates, we expect there to be a reaction force on it, but it turns out that the radiation reaction force is zero for uniform acceleration. This can be shown either from considerations of energy and momentum conservation [4], or by calculating the electromagnetic force an extended charge distribution exerts on itself when accelerated [3].

Despite these arguments, it is generally agreed that the uniformly accelerating charge does in fact radiate, and we can of course calculate a radiation rate. That was the conclusion of Bondi and Gold in a paper they published over 50 years ago [5], but that raised another problem for them which is best introduced by a quote:

> The principle of equivalence states that it is impossible to distinguish between the action on a particle of matter of a constant acceleration or of static support in a gravitational field. This might be thought to raise a paradox when a charged particle, statically supported in

> a gravitational field, is considered, for it might be thought that a radiation field is required to assure that no distinction can be made between the cases of gravitation and acceleration.

So now we are talking about a gravitational field and an equivalence principle, and we are concerned about whether a static charge in a gravitational field should be able to radiate. This can be spelt out in the following way.

A static homogeneous gravitational field (SHGF) is usually modelled in general relativity (GR) by a metric interval of the form

$$\mathrm{d}s^2 = \left(1 + \frac{gy^1}{c^2}\right)^2 (\mathrm{d}y^0)^2 - (\mathrm{d}y^1)^2 - (\mathrm{d}y^2)^2 - (\mathrm{d}y^3)^2 \;, \tag{2}$$

where c is the speed of light and g a constant with units of acceleration. The metric $g_{\mu\nu}$ is almost in the standard Minkowski form, except for the component g_{00}, which is a function of one of the space coordinates y^1.

Now it turns out that this is precisely the SE line element for an eternally uniformly accelerating observer with absolute acceleration g, i.e., $a^\mu a_\mu = -g^2$. So we can show that the curvature is zero and there are therefore no tidal effects, hence the name homogeneous for this spacetime. This is thus a flat spacetime, despite the fact that we are taking it to model a gravitational field, and we can show that there exist coordinates such that the metric assumes the Minkowski form everywhere and everywhen. Those would then be interpreted as the coordinates that would naturally be adopted by a freely falling observer.

So what was Bondi and Gold's problem? They do not believe that a charge sitting at fixed space coordinates in the SHGF should radiate. But the trouble is that general relativity, with a little help, really does predict that it should radiate. And the little bit of help is an equivalence principle. So if we do not believe that a charge sitting at fixed space coordinates in the SHGF should radiate, perhaps it is the equivalence principle that is wrong, or somehow inapplicable to charged particles.

This is therefore a good point to recall the two equivalence principles that form part of any introductory course on general relativity. The first is usually called the *weak equivalence principle* (WEP), although there is nothing weak at all about it. In fact it forms part of the standard formulation of GR for any curved spacetime. We impose the metric condition, which says that the covariant derivative of the metric should be zero, and this fully determines the connection in the torsion-free case. It then turns out that the first coordinate derivatives of the metric components are linear combinations of the connection coefficients, so if we can arrange for the latter to be zero at some event P by a clever choice of coordinates, we will find that the metric components are slowly changing functions of the coordinates at that point.

We then have the following standard argument. For any event P in spacetime, there is always a choice of coordinates in some neighbourhood of that event for which the connection coefficients are zero at P and the metric takes the Minkowski form at P. By continuity and the above observation, this will then be approximately so in some small neighbourhood of P. Basically, WEP thus guarantees the mathematical existence of local inertial coordinates at any spacetime event in the manifold and decrees that these correspond to the coordinates one would naturally set up in a freely falling, non-rotating laboratory.

But we still need to be able to talk about electromagnetism in the framework of a general curved spacetime, and for this we need the *strong equivalence principle* (SEP). This states that, in the locally inertial frames whose existence is guaranteed by WEP, all physics looks roughly as it does in the context of special relativity. This is a rather vague statement and would be difficult to use. In practice, we take the special relativistic formulation of whatever non-gravitational physics it is we are trying to do, e.g., Maxwell's equations if we are doing electromagnetism, and replace all coordinate derivatives by covariant derivatives. At least, this is the simplest or minimal way to implement the strong equivalence principle. There are more sophisticated ways which will not concern us here. This then leads to the minimal extension of Maxwell's equations (MEME) to a general curved spacetime.

Now imagine a charge held at fixed SE space coordinates in an SHGF. It turns out that it is accelerating uniformly, and because of that, SEP tells us that it produces exactly the same fields in the global inertial frame that happens to be available in this case as a uniformly accelerating charge in a gravity-free spacetime. So if there is radiation in the latter case, there will also be radiation for the static charge in an SHGF. This is an application par excellence of the strong equivalence principle in the sense that there is no approximation here due to local effects, since the local inertial frame is globally inertial.

So if we think a static charge in a static spacetime cannot radiate EM energy, then here is another argument against the uniformly accelerating charge in flat spacetime without gravity being able to radiate EM energy. However, as mentioned earlier, the consensus says that it can. Alternatively, 'the' equivalence principle may be wrong. But WEP is built into standard GR, and GR would be virtually unusable without SEP, in the sense that we would require some other way of shipping our non-gravitational theories of physics into the curved spacetime framework. And another alternative is that a static charge in a static spacetime may after all be able to radiate EM energy.

We have now sketched the whole issue here. This is a tangle of at least three problems, probably more:

- Do eternally uniformly accelerating charges radiate EM energy?
- Does this debunk some form of equivalence principle?
- Can a stationary charge in a static spacetime radiate EM energy?

Let us begin by addressing the last question in more detail.

A static spacetime is one in which there exist coordinates in which the metric interval assumes the form

$$\mathrm{d}s^2 = g_{00}(x^1, x^2, x^3)(\mathrm{d}x^0)^2 + \sum_{i,j=1,2,3} g_{ij}(x^1, x^2, x^3)\mathrm{d}x^i \mathrm{d}x^j \;, \tag{3}$$

so that the metric components do not depend on the time coordinate, and in addition the matrix of metric components is in block diagonal form with g_{01}, g_{02}, and g_{03} equal to zero. A static or stationary charge is one that sits at fixed space coordinates in a spacetime with one time coordinate and three space coordinates. This kind of staticity is clearly a coordinate dependent notion. To give an example, a static charge in a semi-Euclidean coordinate system is accelerating.

Now Bondi and Gold say that a static charge in a static spacetime cannot radiate EM energy. However, GR with the help of SEP and MEME predict that a freely falling observer will observe EM radiation from a static charge in an SHGF, if uniformly accelerating charges in gravity-free spacetimes do radiate. And since Bondi and Gold found that uniformly accelerating charges do radiate in the latter case, they had to come up with some other solution. And here it is: there is no such thing as an infinite static homogeneous gravitational field.

The point is that, if we do away with the SHGF, we may try to argue like this. Radiation from a charge can only be established, according to Bondi and Gold, by surveying space out to large distances. At any distance over which one could affirm the observation of EM radiation, the presence of a gravitational field would be revealed by its inhomogeneity. The EM effects do not then have to be the same as in an SHGF, and this is supposed to save the charged particle from having to radiate.

The weak point here is presumably the first claim, that one must be a long way from the charge in order to ascertain whether or not it is radiating. Of course, we must agree that the SHGF is unphysical, but this is all a matter of approximation, and there is no quantitative link between approximations to gravitational effects, which have one kind of source, viz., matter and energy, and approximations to EM effects, which have a quite different kind of source, viz., charges and the motions of charges.

Many people commented on this over the following 25 years, in particular Rohrlich, and we shall return to his views on these matters later. But in 1980, Boulware came up with a complete mathematical analysis of the EM fields due to an eternally uniformly accelerating charge [6], and he concluded that there would be EM radiation. So let us examine his arguments for reconciling these issues.

Boulware is interesting because there is a subtle change of tack here. He does not claim that a charge that is stationary relative to coordinates in which the metric is static will not radiate, only that an observer that is stationary relative to these coordinates will not be able to *measure* any radiation there is. So for a charge supported in an SHGF, there is radiation for the freely falling observer, but not for a co-accelerating observer sitting with the charge. Something is therefore telling him that the co-accelerating observer must not be able to see any radiation.

This seems to raise several questions. First of all, why should anyone want to show that? Here is a conjecture. Suppose I am holding a charged particle and moving inertially. Then I will not be able to tell what velocity I am moving at by looking at the EM fields of the charge. This is because Maxwell's equations

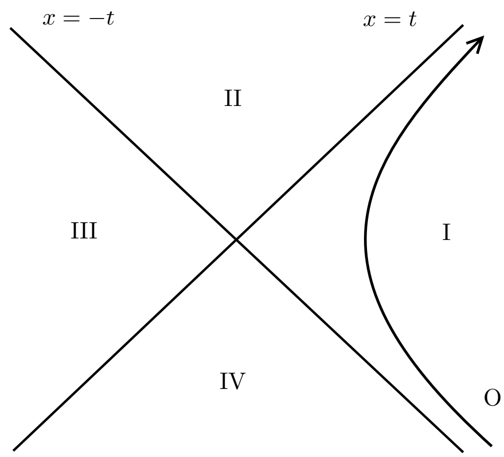

Figure 2: Boulware's four regions of spacetime. The line $t = x$ is an event horizon, because the observer O can never receive any signal from regions II and III on the other side of it. Pictorially this is because the forward light cone of any potential signalling event in region II or III is entirely contained within those regions. For a similar reason, O can never signal to any event in regions III or IV, because the forward light cone of any point on the worldline of O is entirely contained within regions I and II

are Lorentz symmetric. So perhaps the idea here is that I will not even be able to tell whether I am accelerating or not. The point is that I could be sitting still in an inertial frame and holding the charge and not see any radiation, or I could be accelerating uniformly and holding the charge and not see any radiation.

The problem is of course that this fails in the details, because the fields look different in the accelerating case, for any choice of coordinates the observer might make to express those fields. However, that does seem to raise another question: how do we know what accelerating observers will see?

Before returning to this question, let us consider the two arguments Boulware gives to try to support his claim (see the original paper [6] or the detailed discussion in [2, Chap. 15]). Figure 2 shows spacetime again. The cross represents the light cones at the spacetime origin. Now for reasons of causality, the charge can only produce fields in regions I and II, which is $x + t \geq 0$. But also for reasons of causality, if we travel with the charge, we can only get news from regions I and IV. We can never get news of the fields in region II. So we cannot send a friend into region II and he phones later to say that he is witness to some nice radiating fields produced by the charge we are travelling with.

In fact, we are stuck looking at the fields in region I, and Boulware tries to convince us that, if we do that, those fields will look more Coulomb than radiating, that is to say, they will look more $1/r^2$ than $1/r$ for a suitable choice of distance r. However, we should perhaps be asking whether the observer could not accurately predict the fields even into region II from sufficiently accurate measurement of the fields in the close neighbourhood of the charge worldline. And we may also wonder why we should care about what one particular observer can or cannot measure.

Boulware's second argument concerns the generalisation of the Poynting vector to the SE coordinate system, which is identically zero everywhere in region I, as pointed out earlier. This is often cited as definitive proof that the co-accelerating observer could not detect any radiation. But what is the physical meaning of this generalisation of the Poynting vector to coordinates other than inertial coordinates? According to Parrott, one of the main post-Boulware commentators on these issues, it does not give an energy flow at all when integrated over a spacelike hypersurface, but another radiated quantity, associated with a Lorentz boost Killing vector field [7].

The Killing vector fields play an important role in these discussions, so it is worth recalling the basics. A Killing vector field (KVF) is a vector field K such that the Lie derivative of the metric along the flow of K is zero. This is something very convenient from a mathematical point of view owing to the elegant formulation

$$K_{\mu;\nu} + K_{\nu;\mu} = 0 \; . \tag{4}$$

The flow of K is related to a symmetry of the metric, i.e., an isometry, so in a general curved spacetime, there are no Killing vector fields. However, in a static spacetime, there is always at least one Killing vector field, namely the time coordinate vector field for coordinates in which the metric assumes its static form (3).

But what can we do with a Killing vector field? In fact, if we also have a zero-divergence symmetric tensor $T^{\mu\nu}$, i.e., having the properties

$$T^{\mu\nu}{}_{;\nu} = 0 \; , \qquad T^{\mu\nu} = T^{\nu\mu} \; , \tag{5}$$

then we can construct a vector field

$$v^{\mu} := T^{\mu\nu} K_{\nu} \; , \tag{6}$$

and it is straightforward to show that this new vector field will have zero covariant divergence, i.e.,

$$v^{\mu}{}_{;\mu} = 0 \; . \tag{7}$$

It thus represents a conserved quantity, and we can use Gauss' theorem, and so on.

But, of course, the energy–momentum tensor of the EM field is symmetric and divergence-free in the right circumstances, so we can get a divergence-free vector field for every Killing vector field of the metric just by contracting with this energy–momentum tensor. This can be used to define the energy of a field in an inertial frame. The inertial time coordinate vector field in Minkowski spacetime is a timelike, normalised KVF, and it gives the density of field energy–momentum by contracting with the energy–momentum tensor.

Note, however, that not every divergence-free vector field constructed by contracting a KVF with an energy–momentum tensor can be interpreted as a density of field energy–momentum. There is a certain minimal requirement that the Killing vector field must be timelike and normalised at a given event for that to work. This is just the usual intepretation of the energy–momentum tensor in a general curved spacetime. If an observer has four-velocity u, then the contraction of u with the energy–momentum tensor is supposed to give the density of energy–momentum that this observer would measure using standard techniques. And of course, u is a unit timelike vector.

Now Minkowski spacetime is maximally symmetric, so it is absolutely full of Killing vector fields. In fact, it is absolutely full of Lorentz boost KVFs, since there is one in every space direction. Here is the one in the x direction:

$$K(x,t) := x\frac{\partial}{\partial t} + t\frac{\partial}{\partial x} , \tag{8}$$

expressed relative to an inertial coordinate system. Written like this, it may not look much, until we realise that every curve in the flow of this vector field is a uniformly accelerating worldline. Better still, when it is expressed relative to the SE coordinate system for an eternally uniformly accelerating observer in the x direction, it takes on the very simple form

$$K = \partial_\tau , \tag{9}$$

up to a multiplicative constant, where τ is the SE coordinate time. So it is basically the SE time coordinate vector field.

This shows that there is a close relationship between the Lorentz boost Killing vector fields and the SE coordinate systems of eternally uniformly accelerating observers. This is indeed what makes the latter kind of motion so very special in many respects. If we were to consider an observer with arbitrary timelike worldline, we could always construct a SE coordinate system [3, Chap. 12], but the metric would not generally assume the static form (3), and this is because that worldline would not generally be the flow curve of any KVF.

So what about Boulware's second argument, concerning the generalisation of the Poynting vector to the SE coordinate system? According to Parrott, if the SE observer comoving with the charge uses the SE Poynting vector, she will not be calculating a flow of energy at all, but a flow of a kind of pseudo-energy constructed from the energy–momentum tensor and the Lorentz boost KVF by contracting the two. From a Minkowski standpoint, in terms of the flow of energy as it is usually defined in an inertial frame, the calculation of the SE observer looks very strange indeed [2, Chap. 16].

This may be so, but then we know that energy is a frame-dependent concept. We know how to transform the energy of a thing from one inertial frame to another by carrying out a Lorentz transformation of an energy–momentum four-vector, and then we get a different energy for the thing in each inertial frame. But here, we are talking about non-inertial frames and this seems to raise several questions:

- How should an accelerating observer define energy?

- What energy would be measured by an accelerating observer using standard techniques?
- How should an accelerating observer define radiation?

Of course, as a vector, an energy–momentum four-vector can be represented relative to any coordinate or other frame, but here we are suggesting a *different definition* which favours the idea that the relevant quantity should be a conserved quantity.

Before considering these questions from a different angle, let us just do a small detour and examine Boulware's equivalence principle, since we have unfinished business there. A striking thing about many of the papers that purport to be discussing the equivalence principle in this context is that they often give no usable statement of the equivalence principle. We usually have something like this: a uniformly accelerated frame must be indistinguishable from a gravitational field. But this kind of statement is clearly open to the kind of subjective interpretation we get from Boulware. What does *indistinguishable* mean? We are saying here that there might be a radiation field for one observer, but another one must not be able to see it. We must ask whether such an idea is really necessary.

Perhaps we are we just trying to save Bondi and Gold's opinion that a static charge in a static spacetime cannot appear to radiate? But this in turn seems to assume something about what constitutes energy and radiation in non-inertial frames, which brings us back to our earlier question. And such statements of 'the' equivalence principle are to be contrasted with WEP and SEP, which have fully objective definitions and fully mathematical implementations.

Let us return to the questions raised above by a slightly circuitous route. Parrott introduces an interesting idea of accelerating the charged particle by means of a tiny rocket with a tiny fuel tank and a fuel gauge for reading off how much fuel has been used. In gravity-free Minkowski spacetime, since the uniformly accelerating charge is radiating energy which can be detected and used, according to Parrott, conservation of energy suggests that the radiated energy must be provided by the rocket. We might then expect to burn more fuel to produce a given accelerating worldline than we would to produce the same worldline for a neutral particle of the same mass.

So according to Parrott, we have an experimental test to determine *locally* whether EM energy is being radiated or not. The key point here is indeed that such a test would be local. This is to be contrasted with Bondi and Gold's idea that we must be far away from the charge in order to find out whether or not it is radiating. In this case, we simply observe the rocket's fuel consumption.

But now consider a rocket holding the charge stationary relative to SE coordinates in the SHGF. If we burn more fuel to carry the charged particle (than to carry a neutral particle) when accelerating in the gravity-free Minkowski spacetime, we shall burn more fuel to support the charged particle in an SHGF, by an application par excellence of SEP. That is what the theory says if we accept the strong equivalence principle: the mathematics is strictly identical in the two cases.

But Parrott says that the equivalence principle does not apply to charged particles [7]. It is not absolutely clear what he means by the equivalence principle, because there is no clear statement of it in his paper. However, he does claim that local experiments will distinguish a stationary charged particle in an SHGF from an accelerated particle in a gravity-free Minkowski spacetime. And of course he may be right. One day we may be able to do this experiment, and we may find that he was right. In which case, we will know that the strong equivalence principle was not applicable here.

On the other hand, if SEP were not always applicable, we would be in some kind of trouble. How would we use GR? We would need some alternative way to ship our non-gravitational theories of physics into the curved spacetime context. And to save SEP, we need to admit that a stationary charge in an SHGF can radiate EM energy, at least as viewed from a freely falling frame.

But it should be said that, apart from Parrott, nobody seems to disagree with that. For example, Boulware and Rohrlich do not disagree with that. They just do not want the stationary charge in the static spacetime to *appear* to radiate to a comoving observer, for some reason. Recall the conjecture made earlier. Perhaps they consider this to be some form of EP, or an extension of a relativity principle to an accelerating situation. However, neither of these ideas are necessary to the system based on GR with WEP and SEP, and both fail in the details, since the fields in the accelerating case look different in the details for any choice of frame the accelerating observer may choose to represent them.

And the question remains: what will non-inertial observers actually observe? In the gravity-free Minkowski spacetime, if we use the SE Poynting vector, are we calculating radiated energy for some observer? Does that give the energy that would be measured by an accelerating observer using standard techniques, whatever that means? Or is it just a good definition? But if it is good, what is it good for? What are we trying to achieve? And does the value we obtain by calculating with the SE Poynting vector convert properly to the extra fuel that would be needed to accelerate a charged particle?

What should concern us here is that there is no obvious reason why an accelerating observer should adopt SE coordinates. After all, they are just coordinates, despite certain convenient features. They are also artificial in some ways [3, Chap. 12]. For example, the accelerating observer would have to use a rigid ruler, i.e., one satisfying the so-called ruler hypothesis, in order to actually measure the space coordinates in the direction of acceleration.

And why not use a tetrad frame [8]? Recall that the SE coordinate frame *is* a tetrad frame along the worldline, but not off it, and there are many ways to extend it to a tetrad frame off the worldline. But which one should we use to represent the EM fields off the worldline?

Let us now look more generally at the idea of a static charge in a static spacetime, but this time consider a general static spacetime, which may or may not be curved, as specified by the metric interval (3), asking once again whether it is true that a charged particle that is stationary with respect to the space coordinates in a static spacetime generates a pure electric field in that frame. Parrott gives a neat mathematical demonstration [7] that, if we have a stationary charge relative to coordinates for which the metric is static, there will be retarded field solutions for such a charge with zero magnetic field. Unfortunately, he has to assume that the electric field is static in order to derive the result, which weakens the argument somewhat, but let us gloss over that for these purposes. Then if the retarded field solution is unique, this means that the retarded field solution for such a charge always has zero magnetic field.

What is interesting about this argument is that it is entirely dependent upon the use of SEP and MEME! Of course, how else could we say anything at all about electromagnetism in a curved spacetime context?

Now the time coordinate in a static spacetime provides a timelike Killing vector field for the static metric, and it is not difficult to show that we may assume the KVF to be normalised along any given curve in its flow. But then this KVF gives a conserved quantity in conjunction with the EM energy–momentum tensor by contracting the two together. So perhaps what a stationary charged particle in this static spacetime does not radiate is the pseudo-energy defined as the conserved quantity corresponding to translation by the formal time coordinate in this spacetime. And perhaps we should indeed define this as the energy for an observer sitting at fixed space coordinates in this spacetime. But if that is a good definition, let us not forget to say what it is good for. What are we trying to achieve? Where is the physics?

Going back to the flat spacetime context that we have been discussing here, Parrott agrees with Boulware that there is no radiation of the conserved quantity corresponding to the Lorentz boost Killing vector field. But he says that this is irrelevant to questions concerning physically observed radiation. And indeed, this is the case, until someone fills the physical gaps in these arguments.

Parrott also considers a stationary charge relative to the usual coordinates for Schwarzschild spacetime, which is a static spacetime, and asks whether it will radiate EM energy? Parrott claims that it would not, which is interesting, because this is exactly what one would say in a naive special relativistic (SR) version of gravity in which gravity is just a force. A stationary particle is inertial in SR, so Maxwell's theory says there will be no radiation.

In fact, it is interesting to contrast what GR and SR say about radiation from supported and freely falling charges, because they make diametrically opposite predictions about this. And this is because they make diametrically opposite claims about which of the two cases is actually accelerating. In GR, the supported charge is accelerating and the freely falling charge is not, while in SR, it is the freely falling charge that is accelerating.

But in order to understand the physical implications of a scenario in GR, we must first look at what is happening in the locally inertial frame guaranteed by WEP, and then deduce things about EM fields by applying SEP, since this is the only procedure we have. And when we look at the static charge in Schwarzschild spacetime as it would be described in a locally inertial frame, we find that it is accelerating, so there is then nothing obvious at all about the conclusion that this charge will not radiate. On the contrary, MEME says it will, at least to the freely falling observer.

Then if energy radiates out, and if it is true that this can be detected locally, it is tempting to consider that this must be supplied by whatever is holding the charge up against the gravitational effects. However, the zero radiation reaction in the case of eternal uniform acceleration confuses this issue. Boulware shows that there is a flow of energy in towards the charge in the case of eternal uniform acceleration, suggesting that this originates from the horizon $x + t = 0$ (see the original paper [6] or the detailed discussion in [2, Chap. 15]). However, there is only a field on this horizon (in fact a distributional field) if the charge has been accelerating like this *forever*, whereas self-force calculations suggest that the radiation reaction force on the charge would be zero at any instant of time when it has uniform acceleration.

For an arbitrary, i.e., not necessarily uniform acceleration, there will be a radiation reaction, and we might then be able to argue that the radiated energy is somehow supplied by whatever is pushing the charged particle off its geodesic. It would be interesting to see concise discussions of this point.

Returning to the way energy is redefined for observers following the flow curve of a KVF, the conserved quantity corresponding to the time coordinate KVF in a static spacetime may be the only natural mathematical candidate for a conserved quantity. But is it what we would normally call energy physically? Or is it just a good definition, and if so, with what aim in mind? We ought to remember that mathematical convenience is not sufficient to be sure that we are doing physics, i.e., that we are getting a useful relationship with what is out there.

Furthermore, if a charge is stationary relative to some coordinates we happen to have chosen, we must remember that these are only coordinates. It will not generally be stationary relative to the kind of coordinates we are supposed to use to understand the theory physically, viz., inertial or locally inertial coordinates. And we should remember that GR is very different from SR as regards gravitational effects, since GR builds in an interaction of sorts between gravity and other fields via SEP. This is indeed how light is affected by gravitational effects in GR.

It is interesting to end this discussion by considering what Rohrlich has to say about these matters in his classic book [9], now in its third edition. Here we focus in particular on the way he suggests that we should interpret quantities expressed relative to non-inertial coordinate systems. Let us begin with a quote:

> [An SHGF] is a field whose lines of force are equidistant parallels, such as the gravitational field in the laboratory. It is known that this type of gravitational field can be simulated by uniform acceleration of a neutral particle in Newtonian mechanics and in special relativity. Is this also true for the motion of a charged particle?

So here we have a rather typical statement of an equivalence principle. Let us see how we get on with that.

He begins by presenting a tempting fallacy, and what is interesting here is to try to determine precisely what it is that he considers to be fallacious in the following statement:

> A neutral and a charged particle cannot fall equally fast in an SHGF, because the charged particle will radiate, being accelerated, and thereby lose energy, hence fall more slowly than the neutral particle.

But if the freely falling charge is accelerating, and if this is not the fallacy here, then it looks as though we are doing special relativity. Anyway, his statement of intent is now to prove that a charged and a neutral particle in an SHGF *will* in fact fall equally fast, despite the fact that, according to him, the charged one loses energy by radiation.

Before examining his argument, let us just note that the GR picture is exceedingly simple in this particular case, because the freely falling neutral and charged particles are stationary in the global Minkowski frame. The charged particle will not radiate, so this is a solution of the free particle equation of motion, i.e., we do not need to consider the kind of sophisticated arguments expounded in the classic paper by DeWitt and Brehme [10], which show that curvature can intervene directly in the equation of motion and change the whole notion of free fall for charged particles. Note also that the charge would only radiate in the SR picture!

Returning to Rohrlich's discussion, he thus sets out to prove that a charged and a neutral particle in an SHGF will in fact fall equally fast, i.e., have the same worldline or the same shape of worldline, despite the fact that the charged one, according to him, loses energy by radiation. His argument is basically GR+SEP leading to MEME, but note that he still claims that there is radiation. However, we then discover that the freely falling observer will not see any radiation, and this because the charge is just sitting still in an inertial frame.

So this is precisely the GR picture, and we begin to wonder what we must do in order to see this radiation. In fact, it turns out that we have to be stationary relative to the SE coordinates for the SHGF to see it. However, the field of the charged particle in the freely falling frame is Coulomb, so what we are claiming here is that a Coulomb field will *look like* a radiating one to an accelerating observer, whatever that means. But even if it did, is that how the accelerating observer should understand what is happening, by looking at the electric and magnetic fields relative to some coordinates that happen to be adapted to her worldline?

After all, these are not the only possible coordinates that such a person could use. There are other adapted coordinate systems, and there are tetrad frames that could be used to express the fields off the worldline. But which picture should the accelerating observer use?

We may consider another example of what seems unholy in this account of things. The geodesic equation in the SHGF says precisely that the four-acceleration of a thing is zero, and then we say that the thing is freely falling. Fiddling around with the coordinates will not make free fall in this flat spacetime, or indeed in any other spacetime, become a uniform acceleration, because it is zero acceleration. But what Rohrlich suggests here is that, if the supported observer using SE coordinates should somehow be duped into thinking that her coordinate system has any real significance, the freely falling particle may *appear* to have this or that acceleration. So in this view of things, coordinates can be taken by their resident observer, if there is one, to have some real physical significance.

It is interesting that Rohrlich should seek coordinates for the supported observer relative to which free fall looks like uniform acceleration, because free fall in an SHGF *is* uniform acceleration in the naive special relativistic model of gravity in which gravity is just a force [2, Chap. 3], another striking result concerning uniform acceleration.

Anyway, transforming the Coulomb field in the freely falling frame, which we know to solve MEME in the GR version of the SHGF, Rohrlich claims that we obtain a radiating field in the SE coordinates. So in his view the supported observer will 'see' this charge as radiating. He goes further, giving the standard formula for the radiation rate in ordinary Minkowski coordinates in SR, specifying how it is found algebraically from the components of the EM field tensor relative to such a frame and noting that this rate is Lorentz invariant. But what he then suggests is that, when we transform to arbitrary coordinates for this spacetime (no longer a Lorentz transformation), we can use the same algebraic combination of the new components of the EM field tensor to deliver a rate of energy radiation.

This is just to show how we can get a nonzero rate for the supported observer when the radiation rate is resolutely zero for the freely falling observer. But there are two difficulties here. First of all, the prescription is potentially ambiguous, given the various possible ways of expressing the fields off the worldline. But note that if the radiation rate depends only on a specific point on the worldline, i.e., a specific proper time of the charge, it may be possible to circumvent this difficulty, using the Lorentz invariance of the rate. On the other hand, this leaves us with the problem of interpreting the radiation rate calculated in this way. Is it supposed to be what the accelerating observer would measure using standard techniques, whatever that means, or is it just a good definition? And if so, what is it good for? What are we trying to achieve by it?

Here is an exercise for the reader. Think up an equivalence principle that would make you want the Coulomb field to look like a radiating field to an accelerating observer. One answer is a Newtonian, naive special relativistic, pre-GR kind of equivalence principle which we do not need, and which fails in the details, because this field will never look exactly like the radiating field of an accelerating charge, for any choice of coordinate system the accelerating observer may choose to express the Coulomb field.

However, this is not the end of the mysteries. In 1964, Mould invented an entirely theoretical radiation detector that would bear out such predictions [11]. In other words, when it is moving inertially and there is no radiation, it does not record any radiation, and when it is moving inertially and there is radiation, it records radiation, but when it moves in uniform acceleration past a Coulomb field, it also excites. This is a striking result, if the theory in his paper is correct. There is a close parallel with the Unruh–DeWitt detector in quantum field theory (QFT) (see the appendix).

Rohrlich also asks how we know that a charged particle at rest relative to the supported frame will not radiate. That would be the prediction of an SR version of gravity. In GR, it is perhaps better to say that this charged particle will radiate and that the supported observer can spot this if she wants to! Rohrlich agrees that the freely falling observer will see the supported charge radiating at the well known constant rate. He then transforms the fields to the SE coordinate system and deduces that there is no radiation because the magnetic field is zero in the SE system, an argument we have already discussed twice here.

Such a claim is perhaps best answered by a question: should we treat the SE magnetic field as the kind of magnetic field we know and love from our school days? And let us note once again that, apart from having zero magnetic field and being static, the SE version of these fields does not look anything like the Coulomb field.

Conclusion

This discussion has raised many questions, but two in particular. First of all, how should we formulate our equivalence principles? A good rule might be to stick to WEP and SEP and forget any statements that talk about whether something can be distinguished from something else. The latter are likely to be Newtonian, naive SR, and pre-GR principles that are no longer needed in the GR framework, and liable to fail there.

Another question concerns the way we interpret quantities expressed relative to non-inertial coordinate systems. Here we should perhaps be clearer about whether we are interested in what accelerating observers actually measure, or whether we are just trying to make good definitions for them. But what will accelerating observers observe? What will they consider to be good definitions? And if they are good, what are they good for? What exactly are we trying to achieve? What will accelerating observers measure using accelerating detectors? Indeed, does it help to know what accelerating detectors will detect?

We should remember that there is a major theoretical difference between inertial motion and accelerating motion, both for observers and for detectors. When an observer is moving inertially, we know what are the best coordinates for such a person to use: they are inertial or locally inertial coordinates. This is because all our field theories of matter are Lorentz symmetric or locally Lorentz symmetric, and these are the coordinate systems in which they assume their simplest forms.

Regarding detectors, imagine designing two different detectors to measure the same physical quantity. Whenever they are moving inertially in the same physical context, we expect them to deliver the same value for whatever quantity it is they are supposed to measure. This is once again because all our field theories of matter, which govern both the internal constitution of the detectors and the environment of the detectors, are Lorentz symmetric or locally Lorentz symmetric. But what can we say when they are accelerating? Will they always deliver the same result for the given physical quantity? After all, there is no corresponding acceleration symmetry in our field theories of matter.

Appendix: Unruh Effect

Here is another situation where some quantities expressed relative to a semi-Euclidean coordinate system are interpreted physically as being relevant to uniformly accelerating observers. To illustrate this, we may begin with a quote from an epic review of everything that has been done in this area over the past 40 years [12]:

> [The Unruh effect] has played a crucial role in our understanding that the particle content of a field theory is observer dependent.

This is no longer classical electrodynamics. Here we are talking about quantum field theory, where the concept of particle is subject to a certain ontological fuzziness, and the field is no longer an electromagnetic field, because the Unruh effect is usually introduced by discussing the Klein–Gordon scalar field. Any reference to field equations here can be taken to mean the Klein–Gordon (KG) equation.

If we want to set up a quantum field theory, the key question is: what constitutes a positive frequency solution to the field equations? Once we have that, we can try to make the usual expansion of the field in terms of creation and annihilation operators. But the answer to this question depends on what time coordinate we use. Now in Minkowski spacetime, we normally choose an inertial time coordinate for the QFT construction. But it turns out that we can find solutions to the field equations that are positive frequency with respect to the SE time coordinate for an eternally uniformly accelerating observer. The QFT construction then delivers a different vacuum and different particles.

In inertial coordinates in Minkowski spacetime, the natural positive frequency solutions to the field equations lead to what we normally call particles. Let us call them Minkowski particles for the present purposes. In SE coordinates for an eternally uniformly accelerating observer, associated as we have seen with a Lorentz boost Killing vector field, the natural solutions to the field equations lead to different particles. Let us call these Rindler particles here. How are we to interpret this new Rindler vacuum and these new particles?

The first thing we note is that the usual QFT vacuum, the Minkowski vacuum, is full of Rindler particles. Of course, this is not really so surprising since we have always known that the QFT vacuum is *not* nothing. But we also note that formally, for the right formal choice of 'Hamiltonian', the density operator for the Rindler particles is precisely the density operator for a system of particles in equilibrium at a certain nonzero temperature. This is the Unruh temperature. It is linearly proportional to the absolute acceleration of the eternally uniformly accelerating observer who sets up this alternative view of the quantum field.

But how do we know that the formal 'Hamiltonian', constructed by the usual Lagrangian field theoretical techniques but in the framework of a semi-Euclidean coordinate system, is what an accelerating observer would call energy? Are we saying that this is what such a person would naturally measure, or is it just a good definition? And if it is a good definition, what is it good for? Note that the naturalness of the 'temperature' attributed to the thermal bath of Rindler particles depends on the naturalness of this definition of the Hamiltonian.

Certain features of the new Hamiltonian are not so natural. It comes from a classical field 'energy' for the eternally uniformly accelerating observer, and this classical field energy is defined to be

$$E := \int_\Sigma K^\mu T_{\mu\nu} \mathrm{d}\Sigma^\nu \, , \tag{10}$$

where K^μ is the appropriate Lorentz boost Killing vector field, $T_{\mu\nu}$ is the formal energy–momentum tensor for the KG scalar quantum field, and Σ can be any spacelike hypersurface cutting all timelike curves in the spacetime, since we know that $K^\mu T_{\mu\nu}$ is conserved. A simple choice for Σ is the hyperplane of zero SE time for the chosen observer.

This quantity E is supposed to be the energy of the field as gauged by an observer following a flow line of the KVF. Note that K can be assumed normalised along the observer worldline. However, the expression (10) for E integrates $K^\mu T_{\mu\nu}$ over places where K^μ is not normalised, whence we cannot claim that this quantity is the density of energy–momentum (a four-vector with components equal to the energy density and the rates of flow of energy per unit area in three space directions) that would be measured by the observer following the flow line of the KVF at that particular point.

Worse, all observers following flow curves of the Lorentz boost KVF are accelerating, and we shall soon see that they themselves will not measure the same things with their detectors as instantaneously comoving inertial observers, according to the results of the discussion about detectors below. So this really does look like a case of making the best definition of energy we can, constrained by the requirement that the thing we integrate must be conserved.

Anyway, the usual QFT vacuum is thus described as a thermal bath of Rindler particles at the Unruh temperature. It is usual to joke at this point that, if we accelerate our lunch at high enough acceleration, it will cook, remembering of course not to try to keep up with it! But can we demonstrate that an SE observer will interact with those Rindler particles, just by the fact that she is accelerating? Can we show that such an observer will end up in 'thermal equilibrium' with them? Surprisingly, there are indeed arguments to support such claims.

Things become a lot clearer when we stop talking about observers and start talking about detectors. Consider the Unruh–DeWitt (UD) detector, which is a pointlike detector with a linear interaction with the quantum field and two energy levels (ground and excited). Clearly this was not chosen on the grounds of physical realism, but more sophisticated models have been investigated, and we shall assume that the following discussion is also borne out in more realistic cases.

We can consider this detector in four different situations:

1. Stationary in an inertial frame in the Minkowski vacuum, it does not excite.

2. Accelerating uniformly in an inertial frame in the Minkowski vacuum, it does excite. This case is also described by saying that the detector is stationary, i.e., sitting at fixed SE space coordinates, in the Rindler thermal bath. So we can understand its excitation through absorption of the ontologically fuzzy Rindler particles.

3. Stationary in an inertial frame in a Minkowski thermal bath of the usual (but nevertheless ontologically fuzzy) Minkowski particles at the Unruh temperature corresponding to some eternal uniform acceleration, the detector will excite, but not at the same rate as in case 2.

4. Accelerating uniformly in an inertial frame in the Rindler vacuum, a state of the field in which there are no Rindler particles but which is full of Minkowski particles, the detector will not excite, provided that it has the right absolute acceleration as determined by specifying the motion of the observer who set up this particular Rindler vacuum. (There are different Rindler vacuums, depending on the worldline of the observer who sets them up.) This case is also described by saying that the detector is stationary in the Rindler vacuum, i.e., sitting at fixed SE space coordinates for the observer who set up this Rindler vacuum construction. The striking thing about this case is that the detector does not 'see' the Minkowski particles in the field state, provided it has the right motion.

A large part of the discussion of the Unruh effect concerns interpretation of the eternally uniformly accelerating cases in terms of Rindler particles, or their absence, using the alternative expansion of the quantum field in terms of positive and negative frequency solutions to the KG equation. Note, however, that these expansions are *not* necessary in order to calculate the excitation of the UD detector for arbitrary motion through the Minkowski vacuum. This can always be done using the standard expansion of the quantum field in terms of creation and annihilation operators associated with negative and positive frequency operators for an inertial time coordinate, and in this sense, these results are just standard results about the QFT vacuum.

Cases 1 and 3 above suggest that this detector does function in some sense as a Minkowski particle detector, while cases 2 and 4 suggest that it functions in some sense as a Rindler particle detector, provided that it is doing the right thing in each case, i.e., provided that it has the right motion. Of course, such claims are hampered as always by the ontological fuzziness of the particle notion in quantum field theory.

What about temperature? We can consider not only excitation but also deexcitation of the detector, and we find that the associated rates satisfy a detailed balance relation. If we imagine a 'gas' of these pointlike detectors held at some fixed SE space coordinate, i.e., uniformly accelerating, in the Minkowski vacuum (case 2), and if we consider their excitation and deexcitation rates, we find that their energies will distribute over the two available energies (ground and excited) in precisely the way we would expect for a gas at the Unruh temperature. Likewise for a similar detector 'gas' held stationary in an inertial frame in a Minkowski particle thermal bath at the corresponding temperature, despite the fact that the excitation rates differ in the two cases. So there is some value in this temperature interpretation and it does suggest that one's lunch might indeed cook if accelerated sufficiently.

To end, let us just compare the UD and Mould detectors:

- The Mould detector detects nothing when stationary in a Coulomb field, just as the UD detector detects nothing when stationary in the Minkowski (usual QFT) vacuum.

- The Mould detector detects radiation when accelerated through the Coulomb field, just as the UD detector detects something when accelerated through the Minkowski vacuum.

- The Mould detector detects nothing when accelerated with the charge source, just as the UD detector detects nothing when 'stationary', i.e., uniformly accelerating, in the Rindler vacuum.

- The Mould detector detects something when the charge is accelerated and the detector is moving inertially, just as the UD detector will detect something when moving inertially in a Minkowski thermal bath.

The parallel continues slightly:

- The excitation rate of the Mould detector when accelerated through a Coulomb field is not the same as when it moves inertially through the field of an accelerating charge (and in any case, the field of an accelerating charge does not look exactly like a Coulomb field for any choice of coordinates adapted to the motion of the accelerating charge).

- The excitation rate of the Unruh–DeWitt detector when uniformly accelerated through the Minkowski vacuum with some absolute acceleration a, construed as being stationary in a Rindler thermal bath, is not the same as its excitation rate when stationary in a Minkowski thermal bath at a temperature equal to the Unruh temperature corresponding to the acceleration a.

Those who study the Unruh effect have referred to the whole subject of the discussion about uniformly accelerating charges and the equivalence principle as being merely a semantic issue! But what is physics if not the semantics of our mathematical models, i.e., an attempt to extract meaning from such models in the context of physical measurement in the real world?

In any case, the same could be said about accounts of the Unruh effect. And a problem remains with these accounts: they claim to give the perspective of an accelerating observer, while it is quite clear that this approach only works (only exists) when the accelerating worldline is a flow curve of a Killing vector field. Without this, there is apparently no elegant alternative construction of the quantum field theory.

Note on the Exercise for the Reader

Consider a situation in the naive SR version of gravity, in which gravity is just a force, and we have Maxwell's theory in flat spacetime. Imagine an observer sees a set of particles of various masses all accelerating away with the same acceleration. She may construe this as a situation in which there is a gravitational field and the particles are in free fall while she is held up against fall (case 1), or one in which there is no gravitational field, the particles are moving inertially, and she is accelerating away from the particles (case 2).

A Newtonian EP based on the equality of passive gravitational mass and inertial mass claims that these situations will be indistinguishable to that observer. But what happens if the particles are charged? When they are freely falling, they will generate EM radiation in the inertial frame of the observer, because they are accelerating in this SR picture. But if they are stationary in an inertial frame, they will produce Coulomb fields, and in order for the two situations to be 'indistinguishable', the accelerating observer in this second scenario must 'see' these Coulomb fields as radiating fields.

Unfortunately, whatever coordinates or other frame the accelerating observer uses in the second case, the Coulomb fields of the charges will not look exactly like the radiating fields of accelerating charges to an inertially moving observer, so that would nevertheless fail in the details.

What would be a GR version of this thought experiment? Actually, the two situations can be construed in many ways, but the one to watch is this. If we consider case 1 to be free fall of the particles and the observer supported against free fall, and case 2 to be inertial motion of the particles and acceleration of the observer, then the two scenarios are identical in GR and there is no more to be said about distinguishing them.

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