In scheme B, the two players begin by determining a long series of simulations of θ-strategies for all the different θ in $[0,\pi/4]$. They do this by drawing up pairs of instruction sets with forbidden zones of the form

$$\mathrm{FZ_B} := [0,\theta/2] \cup [\pi/2-\theta/2, \pi/2+\theta/2] \cup [\pi-\theta/2, \pi] \ . \tag{1}$$

So for each trial, there is a pair of identical instruction sets, one for each player, characterised by an angle θ. Call that a θ-trial. In each trial, the polarisers are set and there will be some misalignment α between them, anywhere between 0 and $\pi/2$. We assume that α is evenly distributed over $[0,\pi/2]$. If one polariser falls in $[\theta/2,\pi/2-\theta/2]$ and the other in $[\pi/2+\theta/2,\pi-\theta/2]$, we record disagreement. If both fall in $[\theta/2,\pi/2-\theta/2]$ or both fall in $[\pi/2+\theta/2,\pi-\theta/2]$, we record agreement. If either falls in $\mathrm{FZ_B}$, we record nothing.

For each θ, we have to establish an actual frequency $A_\mathrm{B}(\theta)\mathrm{d}\theta$ with which to carry out θ-trials for this value of θ, satisfying

$$\int_0^{\pi/4} A_\mathrm{B}(\theta)\,\mathrm{d}\theta = 1 \ .$$

It seems that no choice of $A_\mathrm{B}(\theta)$ can solve the problem because it looks as though we can only simulate a linear target function over $[0,\pi/4]$, although it may be possible to simulate more general functions over $[\pi/4,\pi/2]$. This is my analysis.

We know the probability that a θ-trial with misalignment α will be recorded, viz.,

$$E_\theta(\alpha) = \begin{cases} \dfrac{\pi-2\theta-2\alpha}{\pi}\ , & 0<\alpha<\theta\ ,\\[2mm] \dfrac{\pi-4\theta}{\pi}\ , & \theta<\alpha<\dfrac{\pi}{2}-\theta\ ,\\[2mm] \dfrac{2\alpha-2\theta}{\pi}\ , & \dfrac{\pi}{2}-\theta<\alpha<\dfrac{\pi}{2}\ . \end{cases} \tag{2}$$

This has average

$$E_\theta^\mathrm{ave} = \frac{2}{\pi}\int_0^{\pi/2} E_\theta(\alpha)\,\mathrm{d}\alpha = \frac{(\pi-2\theta)^2}{\pi^2} = \left(1-\frac{2\theta}{\pi}\right)^2\ , \tag{3}$$

provided there are enough trials for the misalignment to effectively sample the whole of $[0,\pi/2]$ for the given value of θ and assuming that the values of α are chosen completely randomly in $[0,\pi/2]$, i.e., they are uniformly distributed. This average is the probability that the result of a θ-trial will be recorded, regardless of the value of the misalignment.

So $A_\mathrm{B}(\theta)$ is the frequency distribution of θ-trials agreed by the two players before the experiment begins. We wish to calculate the long run proportion of agreement (POA) for some particular value of α in $[0,\pi/2]$, and consider first a value of α less than $\pi/4$. For a long run of N trials ranging over all $\theta\in[0,\pi/4]$, we have a total of

$$A_\mathrm{B}(\theta)N\mathrm{d}\theta$$

with θ in $(\theta, \theta + \mathrm{d}\theta)$, and we expect

$$A_{\mathrm{B}}(\theta) N \mathrm{d}\theta E_\theta(\alpha) \tag{4}$$

of these to be recorded, provided that the number $A_{\mathrm{B}}(\theta) N \mathrm{d}\theta$ is big enough. This follows because $E_\theta(\alpha)$ is precisely the fraction of θ-trials that get recorded for this value of the misalignment.

Now when $\theta > \alpha$, we know that the players always agree, while for $\theta < \alpha$, they agree a proportion

$$y_\theta(\alpha) = \frac{\pi - 2\theta - 2\alpha}{\pi - 4\theta}$$

of the time, so the long run expected number of agreements in recorded trials will be

$$\int_0^\alpha y_\theta(\alpha) E_\theta(\alpha) A_{\mathrm{B}}(\theta) N \mathrm{d}\theta + \int_\alpha^{\pi/4} E_\theta(\alpha) A_{\mathrm{B}}(\theta) N \mathrm{d}\theta \,.$$

We require this to equal $N f(\alpha)$, where $f(\alpha)$ is the target function for the POA, which would be $\cos^2\alpha$ in the quantum case.

Looking at (2) and noting that $\alpha < \pi/2 - \theta$ since $\theta < \pi/4$ and $\alpha < \pi/4$, we see that in the first integral $\theta < \alpha$ and we can insert

$$E_\theta(\alpha) = \frac{\pi - 4\theta}{\pi} \,.$$

For $\theta > \alpha$, as happens in the second integral, we can insert

$$E_\theta(\alpha) = \frac{\pi - 2\theta - 2\alpha}{\pi} \,.$$

Hence the long run expected number of agreements in recorded trials will be

$$\int_0^\alpha \frac{\pi - 2\theta - 2\alpha}{\pi - 4\theta} \frac{\pi - 4\theta}{\pi} A_{\mathrm{B}}(\theta) N \mathrm{d}\theta + \int_\alpha^{\pi/4} \frac{\pi - 2\theta - 2\alpha}{\pi} A_{\mathrm{B}}(\theta) N \mathrm{d}\theta \,,$$

and this can be written in the form

$$\frac{N}{\pi} \int_0^{\pi/4} (\pi - 2\theta - 2\alpha) A_{\mathrm{B}}(\theta) \mathrm{d}\theta \,.$$

The long run expected POA is thus linear in α. It can be written

$$1 - \frac{2}{\pi} \int_0^{\pi/4} \theta A_{\mathrm{B}}(\theta) \mathrm{d}\theta - \frac{2}{\pi}\alpha \,.$$

This is indeed always between 0 and 1 because the middle term is 2π times the average value of θ over all θ-trials (regardless of whether recorded or not), hence lies between 0 and $(2/\pi) \times (\pi/4) = 1/2$, while $2\alpha/\pi$ also lies between 0 and 1/2 for α in $[0, \pi/4]$.

It is interesting to see what we can simulate for the POA when $\alpha \in [\pi/4, \pi/2]$. Since θ lies in $[0, \pi/4]$, we have $\theta < \alpha$ for all θ-trials. When θ is such that

$\pi/2 - \theta < \alpha$, i.e., for θ such that $\theta > \pi/2 - \alpha$, we know that we always have disagreement, so there is no contribution from these values of θ. We only need to consider θ such that $\pi/2 - \theta > \alpha$, i.e., θ such that $\theta < \pi/2 - \alpha$, noting that $\pi/2 - \alpha$ is always less than $\pi/4$. In these trials, the players agree a proportion $y_\theta(\alpha)$ of the time.

We just have to include the fraction of cases $E_\theta(\alpha)$ when the results are actually recorded. Looking at (2) on p. 1, we note that we are always in the case $\theta < \alpha < \pi/2 - \theta$, when the proportion of results recorded is $(\pi - 4\theta)/\pi$. So the long run expected number of agreements in recorded trials, for a very long run of N θ-trials ranging over all values of θ, is

$$\int_0^{\pi/2-\alpha} \frac{\pi - 4\theta}{\pi} \frac{\pi - 2\theta - 2\alpha}{\pi - 4\theta} A_{\mathrm{B}}(\theta) N \mathrm{d}\theta \ ,$$

the first factor in the integrand being the relevant value of $E_\theta(\alpha)$ and the second being $y_\theta(\alpha)$. This can be rewritten

$$\frac{N}{\pi} \int_0^{\pi/2-\alpha} (\pi - 2\theta - 2\alpha) A_{\mathrm{B}}(\theta) \mathrm{d}\theta \ .$$

This will not generally be a linear function of α, and it may be possible, with ingenuity, to match it to any target function $f(\alpha)$ over $[\pi/4, \pi/2]$. Note also that we never need to know the values of $E_\theta(\alpha)$ in the range $\alpha \in [\pi/2 - \theta, \pi/2]$.