The Quantum World

Stephen N. Lyle

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In what sense is quantum physics a *quantum leap* away from the classical physics that preceded it? This theory, or rather this family of theories, has raised a good many questions, including philosophical questions:

- Is the world fundamentally *indeterminist*? Or is everything that happens in the Universe already predetermined?
- Can the human mind act directly on matter to influence its behaviour? This would be like a kind of telekinesis, an idea long abandoned by science.
- And what about holism? Is everything in the Universe somehow linked to everything else at the most fundamental level in some kind of entangled state? This would be a kind of opposite to the idea of reductionism.

We shall have time to return to some of these questions.

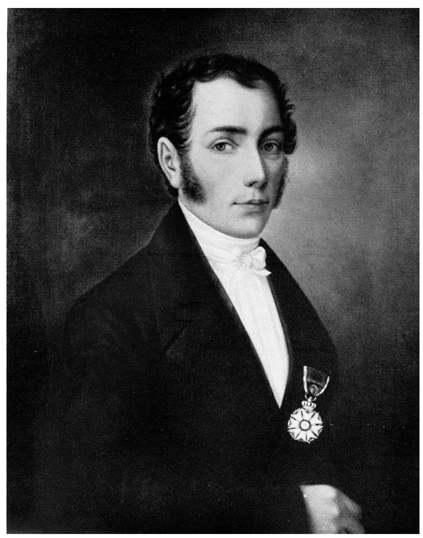

Figure 1: Josef von Fraunhofer (1787–1826). Source: en.wikipedia.org/wiki/Joseph_von_Fraunhofer

But first, we need to get some idea of what this theory is about. Before the philosophy, we need to have the facts! The following presentation will be divided into three parts:

- Why did scientists invent the quantum theory?
- Wave–particle duality
- Quantum nonlocality

So let's begin at the beginning. When we invent a theory, it's because we've observed something that calls out for an explanation. So what were those observations?

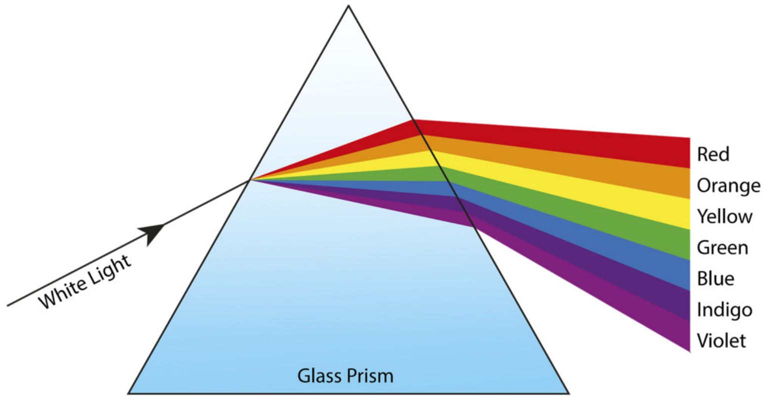

Figure 2: When white light goes through a prism, it splits into all the colours of the rainbow. Source: biglasers.com

Our story will begin in 1814 with a certain Josef von Fraunhofer, a Bavarian physicist who also made optical lenses (see Fig. 1). Fraunhofer passed a beam of sunlight through a prism. Today, everyone knows what that does (see Fig. 2). Fraunhofer also knew perfectly well, but he did something else as well. We'll see what that was in a moment. The first thing shown by this experiment is that white light is somehow a mixture of colours. That may seem a bit strange. Anyone who has children will know that, when we mix paints of different colours, it gives something more like black! But of course, light is not made by mixing paints of different colours. It is in fact made of electromagnetic waves.

It will be useful here to spend a little time on waves, to prepare the ground as it were, because of course they will be needed when we come to speak of the quantum world. Everyone is familiar with waves on the surface of water, whether they be waves on the ocean or ripples on a pond when we drop a small stone into the water. But there are also waves in the air, sound waves. In both cases, there are oscillatory motions of matter and a transfer of energy. Let me explain.

Consider first a ripple on the surface of a pond. Watch as the ripple passes a small leaf floating at the surface. The water lifts the leaf, then puts it back down where it was. The ripple doesn't actually push the leaf forward. This is because the water in the wave is not moving forward either. The only thing moving forward is energy, the energy to lift up another leaf a little further on. You may have had this same experience yourself when swimming in the sea. The waves merely lift you up and then lower you down again, unless of course they are breaking on the shore!

Think about the sound you hear when I speak. My vocal cords make the air vibrate. The air molecules move back and forth on short, fast cycles, passing the effect to molecules a little further away, which then do the same, transferring energy on and on in all directions right up to your ear. But no air is actually

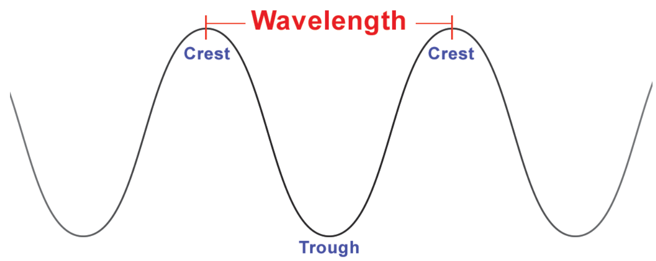

Figure 3: Definition of wavelength. Source:
www.nasa.gov/audience/forstudents/k-4/dictionary/Wavelength.html

transferred from my mouth to your ear. So once again, the matter engages in oscillatory motions, but energy is transferred. We shall see that quantum waves are rather different.

Naturally, there are some technical terms when one wishes to describe a wave. First, the *wavelength*, illustrated in Fig. 3, which is the distance between two wave crests, or indeed between two wave troughs, which comes to the same. The wavelength is generally denoted by the Greek letter λ (pronounced 'lambda'). The physical unit for this quantity is any unit of length, e.g., nanometers, meters, etc. Another important notion is the *frequency*, usually denoted by the Greek letter ν (pronounced 'nu'), which is the number of oscillations of the wave at a given point every second, e.g., how many times per second the leaf on the pond is lifted up, lowered, and brought back to its initial position. The physical unit for this quantity is the hertz (Hz). If a wave oscillates ten times a second, it has a frequency of 10 Hz. With these symbols, a little thought shows that the speed at which energy is transferred by the wave will be the product of the wavelength and the frequency, i.e., $v = \nu\lambda$.

But let's get back to light. It turns out that light can be considered as a wave. Visible light, which is light that can be detected by our eyes, has extremely short wavelengths, in the range from 390 to 750 nm, where

$$1 \text{ nm} = 1 \text{ nanometer} = 10^{-9} \text{ m}\,,$$

or a billionth of a meter. On the other hand, it has very high frequencies, in the range from 430 to 770 THz, where

$$1 \text{ THz} = 1 \text{ terahertz} = 10^{12} \text{ Hz}\,,$$

or a thousand billion hertz. So the wavelengths are just a few thousand times bigger than an atom, and the waves shake at a few thousand billion cycles a second! Among all the colours of the rainbow, red light has the longest waves and the lowest frequencies, while violet light has the shortest waves and the highest frequencies.

I said above that light is made of electromagnetic waves. But what is an electromagnetic wave? What is shaking? One of the most striking things about light is the fact that it can cross a vacuum. It's obvious, in fact, because we can see the stars, the Sun, and the Moon, and space is a vacuum! Such is not the case for waves on water, which really need the water to exist, and waves in the air, which really need the air to transfer their energy. I recall a wonderful experiment we did at school. We put a bell under ... well, a bell jar, i.e., a transparent thing like a bowl, upside down, and hermetically sealed. When there was air under the bell jar and we shook the whole thing, we could hear the bell ringing loud and clear, and of course, we could see it, too. Both sound and light managed to reach us from the bell. But when we extracted all the air from under the bell jar and shook the whole thing, we could no longer hear the bell ringing. But we could still see it moving! Note that sound can also travel through water, and through solids, but it really does need a medium, in stark contrast to light.

For a long time it was assumed that there must be some kind of medium that carried these waves of light. Everyone called it the *ether*. However, no one was ever able to detect it! On the other hand, we may understand the idea of the light wave if we accept the notion of force field. This is because electricity and magnetism both operate via force fields quite capable of reaching receptive objects through empty space. For example, the electron is a particle that carries an electric charge, and we may think of this charge as producing a field of force around the particle. What that means is that, when there is another particle nearby, one that itself carries an electric charge, like another electron, for instance, it will feel the electric force of the first, a repulsion in this case, transmitted to it through this field of force. What's more, when the first electron is shaken from side to side at a certain frequency, that will produce vibrations in the field of force it produces, and those vibrations will be electromagnetic waves with the same frequency as the electron was shaken! In short, light is just a vibration in an electromagnetic force field.

Figure 4 shows all the many possibilities for an electromagnetic wave. Note that radio waves, microwaves, infrared, ultraviolet, X rays, and gamma rays are all electromagnetic waves, just like the light waves we detect with our eyes. Indeed, the part of the spectrum we call 'visible' is just a tiny fraction of all these possibilities. It is instructive to ask *why* our eyes only actually detect this part of the spectrum and none of the other types of electromagnetic waves. One reason is that our star, the Sun, emits particularly strongly in this range, and another is that much of its emissions in other parts of the spectrum are actually blocked out by the Earth's atmosphere and never reach our immediate environment. Naturally, our eyes evolved to be able to detect those electromagnetic waves that were the most freely available and the most useful to our ancestors.

There is something important to understand here. The various waves in the spectrum of Fig. 4 all have different wavelengths and also different frequencies. These are indicated in meters and Hz in the figure. This is indeed precisely what characterises them. Radio waves have the longest wavelengths and the lowest frequencies. Infrared waves are a little longer than visible waves, while ultraviolet waves are a little shorter. In the visible part of the spectrum, red waves are the longest and have the lowest frequencies, while violet waves are

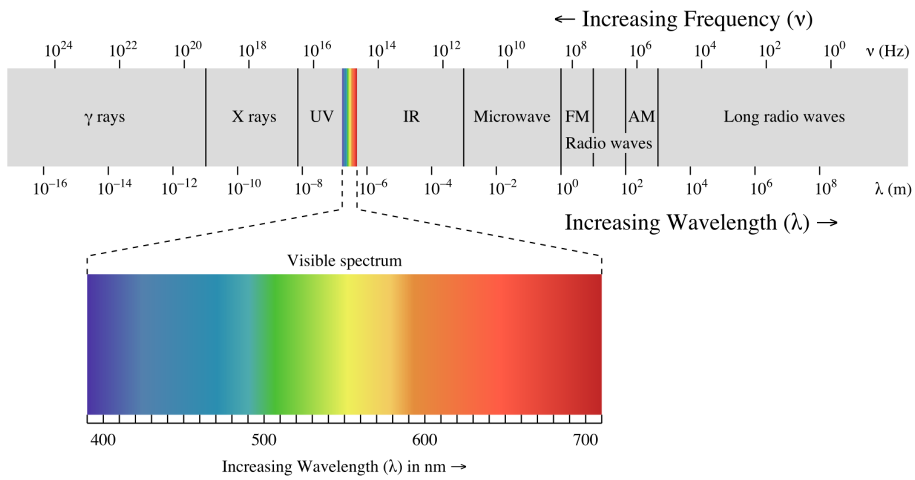

Figure 4: The electromagnetic spectrum. Credit: domain public

the shortest and have the highest frequencies. Each colour, or rather each tint of each colour, corresponds to a precise wavelength and a precise frequency.

Note that the wavelength of an electromagnetic wave is inversely proportional to its frequency. This is because the product of these two quantities is equal to the speed at which energy is transferred by these waves, and this is always the same, because it's the speed of light! And since we're speaking about energy, there's one more important thing you need to know here: each tint of each colour, hence each wavelength and each frequency, corresponds to a precise energy. We shall see in a moment why this is important.

But first take a look at Fig. 5. It shows part of the visible spectrum of the Sun. Fraunhofer analysed a beam of light from the Sun by means of a prism, just as many others had done since Newton's famous experiments in the seventeenth century. But he did much more than that. He examined the spectrum *very closely*, and what he noticed was that certain colours, indeed certain tints of each colour, were missing! Put another way, there were dark lines in the spectrum! Fraunhofer counted 574 dark lines in the spectrum of the Sun's light. Today, looking even more closely, we know of more than 20 000 of these so-called *absorption lines*. Indeed, it's not so easy for light produced in the Sun's interior to actually escape from it. The Sun is surrounded by a gaseous envelope, and what these observations show is that these gases absorb certain frequencies, and hence certain very precisely defined tints of certain colours.

So how can we explain that? To get a better understanding, we need to talk a little about chemistry. The nineteenth century was clearly the century of the new science of chemistry, although of course there had been chemists around for some time, like Lavoisier in France at the end of the eighteenth. One of the main tasks at the time was isolating and identifying new substances, indeed new elements. Chemists like Robert Bunsen and Gustav Kirchhoff in Germany

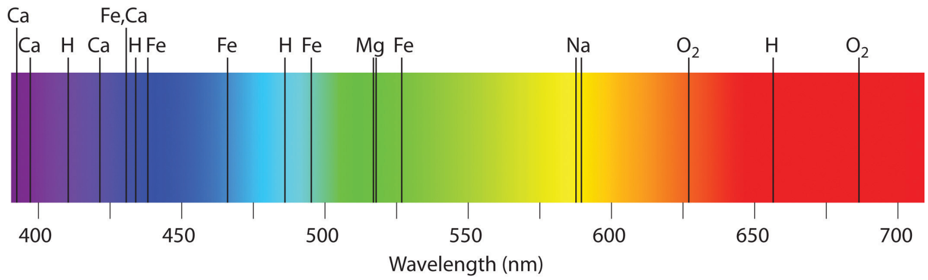

Figure 5: The Sun's spectrum. The dark lines are the tints of colour that are not observed in the Sun's light because they are absorbed by the gaseous envelope around the Sun. They are known as *absorption lines*. The wavelengths are given in nanometers at the bottom. Source: sustainableskies.org/full-spectrum-solar-generated-hydrogen/

would burn these elements and look very closely at the light they emitted, just as Fraunhofer had done with sunlight. What they saw was that each element emits only a selection of very precisely defined tints of colour. The spectra they produced thus consisted of several bright lines on a dark background. It turned out that these sets of *emission lines* were like a kind of fingerprint that characterised the given element. In fact, this is how astronomers can identify the elements present in space or around distant stars, for example, even though they may be very distant. Such a study is called *spectroscopy*. Kirchhoff and Bunsen were effectively identifying the emission lines of different elements.

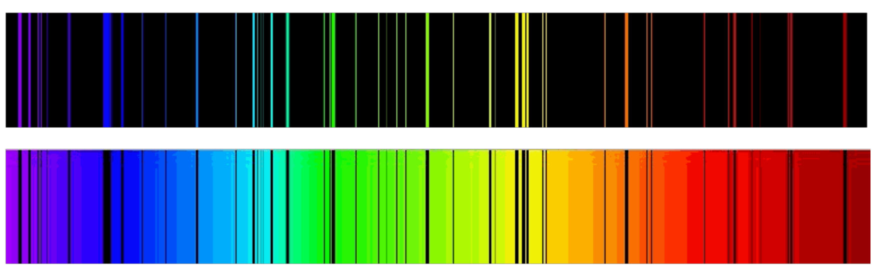

Figure 6: Emission (*top*) and absorption (*bottom*) spectra of mercury. Clearly, this element emits the same tints of colour when it is heated as it absorbs when light passes through it. If we superpose the two spectra, we obtain a continuous spectrum. Source: d.ruze.free.fr/p2dtp/tp6/tp6.htm

So here was a series of observations that really cried out for an explanation. A glance at Fig. 6 shows that mercury emits the same tints of colour when heated as it absorbs when light is passed through it. Figure 7 shows another example, sodium in this case, where we see exactly the same thing. And this was one of the main motivations for inventing the quantum theory: to explain

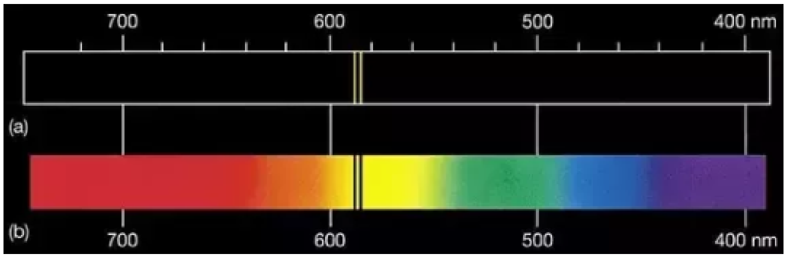

Figure 7: Emission (*top*) and absorption (*bottom*) spectra of sodium. As for mercury, sodium emits the same tints of colour when heated as it absorbs when light is passed through it. If we superpose the two spectra, we obtain a continuous spectrum. Source: www.information-book.com/physics/spectroscopy/

this interaction between light and matter. Because that is indeed what was being observed, an interaction between light and matter.

We have seen how one can understand light (and we shall return to that shortly), but what about matter? Well, we know it's made of molecules, and that molecules are made of atoms. We are still in the first part of this presentation as introduced at the beginning, but we have come to the stage where we can actually introduce the quantum theory. The absorption and emission lines in the spectra of chemical elements call out for an explanation, and that was indeed the reason for developing the quantum theory. Because as we shall see now, the quantum theory is first and foremost a theory about the structure of the atom.

The notion of the atom was reintroduced toward the beginning of the nineteenth century after the work of Lavoisier, Dalton, and others. At first, it was just a building block with no internal structure, although of course this was because there was no way to know otherwise in those days, technically speaking. But at the very end of the nineteenth century, ways were found to extract even smaller particles from atoms. These particles each carried a negative electric charge and were called *electrons*. It was J.J. Thomson who discovered the electron, in Cambridge in 1899, and he also made a theoretical model for the atom, endowing it with an internal structure (see Fig. 8).

Obviously, being British, Thomson allowed himself to be inspired by one of his culinary favourites, and his model survived for a few years, until 1910. Note that the holly berries on top of the pudding in Fig. 8 are not what is supposed to represent the electrons. The idea is better illustrated in Fig. 9. In fact, the electrons correspond to the raisins distributed more or less uniformly throughout the delicious interior of the pudding, which for its part must carry a positive electric charge to balance the negative electric charge of the electrons, since the total electric charge on any atom is always zero, i.e., an atom is always electrically neutral. Presumably, this was the simplest model one could have thought up.

But then in 1909, along came Ernest Rutherford to work with Thomson in Cambridge. Rutherford did an absolutely remarkable experiment which showed

Figure 8: The inspiration for J.J. Thomson's 'pudding model'. Source: www.schaer.com/fr-fr/r/christmas-pudding-pudding-de-noel

that the structure of the atom could not be as Thomson had imagined it. In Rutherford's model, the atom consisted of a small, hard nucleus with a positive electric charge, with the electrons moving around it in some way (see Fig. 9). Rutherford had bombarded gold atoms in a sheet of gold foil with so-called alpha particles, which are actually helium-4 nuclei. Most of these alpha particles went through the foil as if it wasn't there, but from time to time something made them rebound, even coming right back out in the opposite direction. Something hard. The atomic nucleus.

It is important to understand that these entities, the atoms, are really *very* small. On average, an atom will measure something like 10^{-10} m in diameter, which means that one would need ten billion or so sitting side by side in a row to make something a meter long. But the nucleus is *much* smaller again. It measures only 10^{-15} m across, so it's a hundred thousand times smaller than the atom itself, which makes it only a millionth of a billionth of a meter in diameter. To understand this difference, suppose we represent the nucleus by a golf ball, with a radius of about ten centimeters. Then the electrons can wander around over a radius of ten kilometers! And between them and the nucleus, there's nothing. Except, of course, the electric force field. For its part, the electron is so small that we don't even know how small it is, but we do know that it's at least a thousand times smaller than the nucleus.

In 1911, the Danish physicist Niels Bohr arrived in Cambridge to work with Rutherford. Bohr succeeded in making a theoretical model of the structure of the simplest atom, the hydrogen atom. It's the simplest because there is only one electron in this atom (see Fig. 10). Bohr's model can be considered as the first quantum theory.[1] This theory decrees that the electron in a hydrogen

[1]The story of quantum theory is often begun by talking about Planck around 1900, then Einstein around 1905. Both made hypotheses that could be described as 'quantum' in nature. Personally, I would say that the interesting bit of quantum theory comes with the structure of the atom.

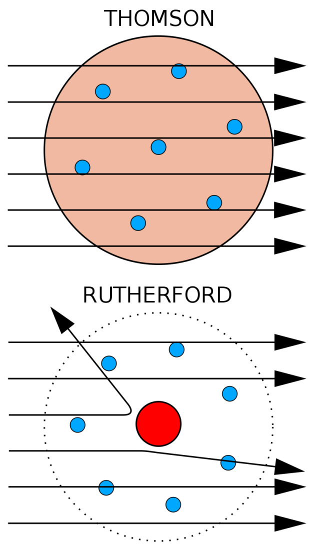

Figure 9: *Top*: Thomson's pudding model. *Bottom*: Rutherford's model, showing how most of the alpha particles (black arrows) are able to go through barely perturbed, unless one of them happens to bump straight into a nucleus (red blob), in which case it rebounds. Source: large.stanford.edu/courses/2017/ph241/sivulka2/

atom cannot have just any 'orbit' around the nucleus. In the hydrogen atom, there is a series of *discrete* energies available for its electron. What's more, they can no longer really be considered as 'orbits'. This is the first insight into a microscopic world that is rather different from the world as we know it at our own macroscopic scale. One of the most important things to note is this contrast between the *continuous* classical world, where every value of a physical quantity is possible over some continuous range, and a world where only a discrete series of values is accessible.

Now, according to this theory, light can displace the electron in the atom from one energy level to another, said to be an *excited level*. Or indeed, the electron can spontaneously drop down from an excited energy level to a less excited one by emitting some light. So the electron in the hydrogen atom can climb up and down a kind of energy ladder, although it always remains attached to the nucleus of the atom.[2] However, there is a condition: to excite the electron from a lower energy level to a higher one, the light must give it *exactly* the right amount of energy, equal to the difference in energies between the two levels. In other words, it must have exactly the right tint of colour. And this is why the hydrogen atom absorbs only certain tints of colour and not others.

On the other hand, when the electron spontaneously de-excites, i.e., when

[2]Note that this does not change any of the chemical properties of the atom, i.e., its tendencies to interact with other atoms.

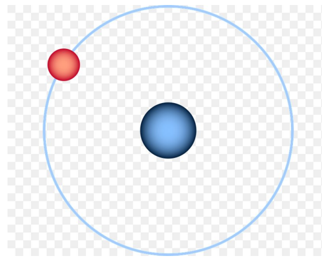

Figure 10: The hydrogen atom. A single electron (red blob) moves around a nucleus (blue blob) which carries a positive electric charge. The whole thing is electrically neutral, i.e., the total electric charge is zero

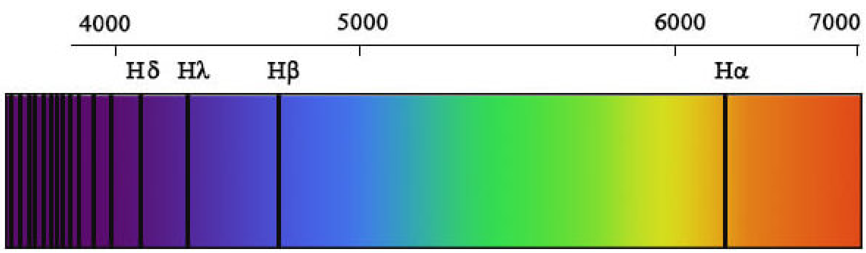

Figure 11: The absorption spectrum of the hydrogen atom which was explained by Niels Bohr's quantum model. The dark lines are the specific tints of colour absorbed by the hydrogen atoms. Numbers indicate the wavelength in angstrom units, where one angstrom unit is a ten billionth of a meter or a tenth of a nanometer. Source: slideplayer.com/slide/1674258/

it falls from one energy level to a lower one, it must emit light at *exactly* the energy corresponding to the difference in energies between the two levels. So it will emit light with the colour corresponding to that energy difference, the same colour as would have been absorbed in order to raise the electron from the lower level to the higher one. So this is why the hydrogen atom emits the same colours of light when excited as it absorbs when light is passed through it.

It remains to explain why the electrons in an atom cannot just do anything they please. Only certain energies are allowed and the explanation eventually put forward was that electrons have a wavelike behaviour. Now you know why we invented the quantum theory and we have come to the second part of this presentation. Here we shall discuss the wavelike nature of the quantum world, and in particular the celebrated *wave–particle duality*. It was actually Louis Victor de Broglie, prince, then Duke de Broglie, a French nobleman, who introduced the idea of matter waves. Later on, he will be one of the heros of this presentation!

But how does it help us to model the electron by a wave? How could that help us to predict discrete values for physical quantities, like the energy

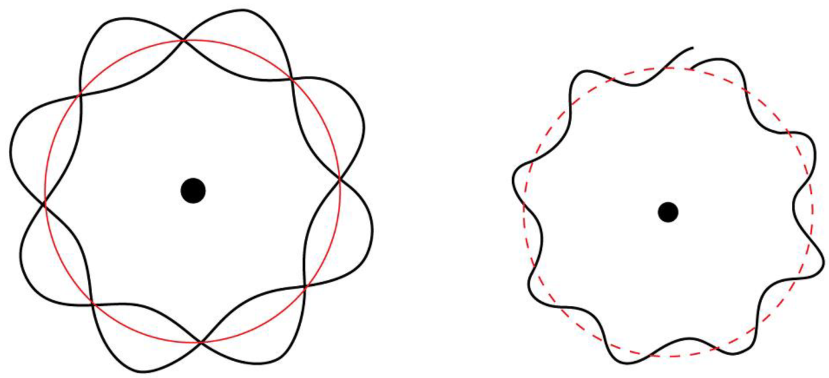

Figure 12: The electron as a matter wave. *Left*: The electron is represented by a stationary wave which follows an orbit around the nucleus of an atom. If the wavelength is fixed, only certain orbits will have the right length to ensure that the wave closes correctly on itself when it has made a complete round trip. *Right*: A forbidden orbit, i.e., one that doesn't have the right length as compared with the wavelength of the electron. Source: www.thestargarden.co.uk/Matter-waves.html

levels of the electron in the hydrogen atom? Hopefully, Fig. 12 gives the idea directly! Everything is explained in the figure caption. The reader wishing to understand more deeply should investigate the notion of *stationary wave*, but note that some slightly more sophisticated mathematics is needed for the actual quantum theoretical version of this.

And now for another wavelike behaviour of matter, one that was first observed for light a long time ago! Let us begin by discussing the experiment with light. It will illustrate a much older problem of wave–particle duality in physics. I'm talking about Thomas Young's two-slit experiment. This time we have to go back even further into the past, to 1801. Young (1773–1829) is sometimes described as the last man who knew everything! (I'm not jealous, in those days there was much less knowledge around than there is today!) To begin with, by the age of fourteen, he could speak English, French, Italian, Latin, Greek, Hebrew, Chaldean, Syriac, Samaritan Aramaic, Amharic (don't worry, I don't know where they speak that either), Turkish, Arabic, and Persian. He studied medicine in London, Edinburgh, and Göttingen, where he obtained the degree of doctor of medicine in 1796. He spent three years in Cambridge before setting up as a doctor in London. Young was appointed professor of natural philosophy at the Royal Institution, where he gave 91 lectures in two years. Then, being a linguist, he was one of the first to begin the decipherment of Egyptian hieroglyphs, even contributing to the translation of the Rosetta Stone. Enough said (and many thanks to Wikipedia for the above).

So let's get back to the two-slit experiment, illustrated in Fig. 13. Using a laser to produce a very fine beam containing exactly one tint of colour, i.e.,

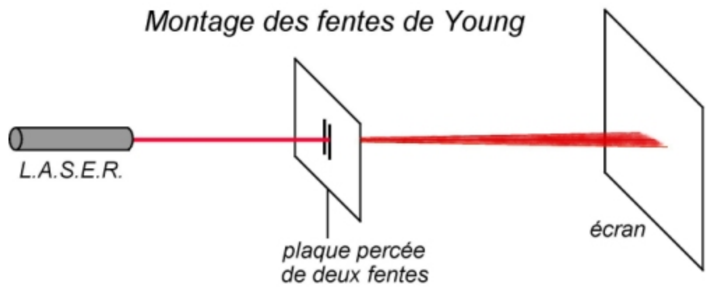

Figure 13: Modern version of the two-slit experiment using a laser to produce a monochromatic beam, i.e., containing only one tint of colour, or one wavelength. The screen on the right is a detection screen. Source: lewebpedagogique.com/physiquempmantes/files/2017/03/TP2_fentes_young.pdf

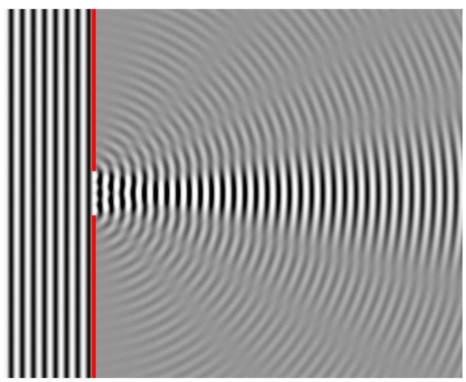

Figure 14: Experiment with a single slit. Diffraction of ripples on the surface of water in a ripple tank. The width of the slit is equal to 4λ, four times the wavelength of the ripples. Image public domain

just one wavelength, we illuminate a screen in which two parallel slits have been cut rather close together. Behind this first screen, there is another, a detection screen (on the right of the picture). So what would you expect to see on this detection screen?

First of all, what would happen if there were only one slit? This is illustrated for water waves in a ripple tank in Fig. 14. As an aside, many wave phenomena can be demonstrated with this simple device, a useful thing to have in the school physics classroom. Here we see the phenomenon called *diffraction*, discovered by Francesco Grimaldi in 1660. Note that the slit must have a width of the same order as the wavelength, which is the case here, because it is equal to four times the wavelength. It's as though a circular wave were generated at the slit, or the slit were the source of such a wave. This effect can also be demonstrated with light. In this case, best results are obtained with a light beam of a precise

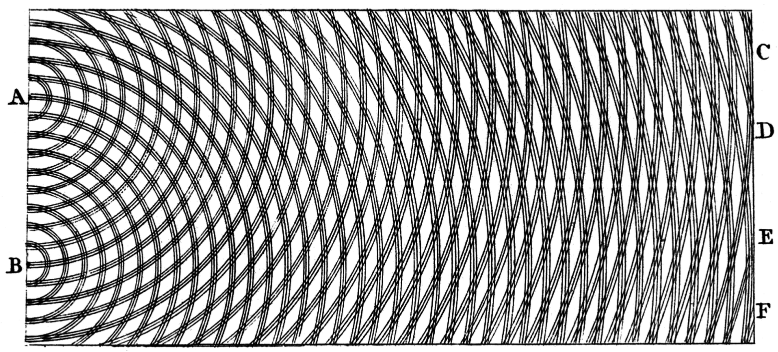

Figure 15: Thomas Young's own sketch of his two-slit experiment using ripples on water. Credit: howthingswork.org/physics-double-slit-experiment-of-light/

tint of colour, produced by a laser, and a slit with width a few hundreds to thousands of nanometers.

But when there are *two* slits, as shown in the setup of Fig. 13, the two circular waves produce interference. Figure 15 shows the sketch done by Young himself. However, drawing may well not have been his strong point, so Fig. 16 shows a more recent image of a simulation of the experiment.

And now we have a particularly important question: what is interference? In fact, it's something everyone knows. If you drop two little pebbles simultaneously side by side into a pond, each will produce its own ripples in growing circles around the points of impact. At some point, these circles will meet . . . then simply cross over one another, their points of intersection generating an elegant pattern. This is the *interference pattern*. This effect confirms what I said earlier about waves: the water itself isn't going anywhere, except up and down. It's the energy in the waves that travels across the surface of the pond, away from the points of impact. Energy which ultimately came from those impacts. And the energy in one ripple can travel straight through the energy in the other.

For its part, the water just goes up and down as a ripple passes. And when two peaks happen to coincide, the height of the water will be the sum of the two heights, or in other words, a superposition. When two troughs coincide, the depth of the water will be the sum of the two depths, another superposition, but negative this time. And when a peak meets a trough, the two can even cancel out altogether. This is what gives the interference patterns in Figs. 15 and 16.

This phenomenon, first observed by Young with light, supported the wave theory of light. In other words, light was not made of little point particles as had been asserted by Newton and Descartes, and as most scientists seemed to think before the beginning of the nineteenth century. However, the debate was raging at the time Young made the above observations, and this subject is highly relevant to our story here, as you will see in a moment. It should be said that, even though it was no simple matter to contradict authorities like

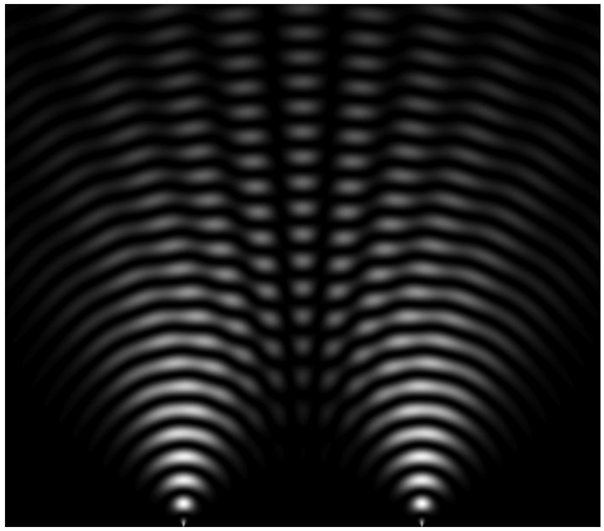

Figure 16: Double slit experiment. The two circular waves produce interference. Image public domain

Newton and Descartes, a wave theory of light had been formulated by another well known scientist, the Dutchman Christiaan Huygens (1629–1695), who had contributed across the board to mathematics, physics, and astronomy. To put it another way, Huygens was also someone to be reckoned with in the world of scientific knowledge.

There is an interesting story relating to this debate. This time in France.[3] In 1807, Thomas Young had just published his experiment with the two slits. A novel experiment was devised about ten years later which came down decisively in favour of the wave theory of light. This was the Arago spot experiment, the very epitome of an *experimentum crucis*. At the time, most of those interested in the question preferred the particle theory of light advocated by Descartes and Newton. This included the noted theoretician Siméon Denis Poisson. In 1818, the French Academy of Science held a competition regarding the paradoxical properties of light. A young engineer, Augustin Fresnel, from the famous *Ponts et Chaussées* school entered the competition. He submitted a paper based on the wave theory of light. One of the members of the jury was Poisson.

Poisson examined Fresnel's theory in considerable detail. In fact, being a partisan of the corpuscular theory, he scoured it for the slightest possible error, or indeed anything that would prove it to be false. And lo and behold, Poisson thought he had found something. One consequence of Fresnel's theory was that a bright spot should form at the center of the shadow cast by an exactly circular opaque body. Needless to say, the corpuscular theory implied that such a shadow

[3]The following is adapted from Wikipedia, which you may consult on the French page fr.wikipedia.org/wiki/Tache_de_Fresnel

should be a uniform disk. Given that one never saw such Fresnel bright spots in everyday shadows, Poisson was sure he had exposed the absurdity of Fresnel's theory.

However, the president of the committee, François Arago[4], decided to actually do the experiment, taking the greatest care over the details. He mounted a metal disk of diameter 2 mm on a glass plate and managed to produce the diffraction spot, for it was indeed another diffraction effect. This experiment was crucial in convincing most scholars that light was indeed wavelike in nature. The prize was duly attributed to Fresnel in November 1819.

Here we have a very good example of the way the scientific 'method' actually works in practice. Each has his preferred idea and sticks to his guns. This is typically human, of course. This was what made Poisson do everything he could to show that Fresnel was wrong. This was what made Fresnel risk everything in pronouncing his preference for a wave explanation of the phenomena at issue. And being a good scientist, Poisson carefully worked out an experimental test. Then, being another good scientist, Arago actually did the experiment, and the whole thing was decided, ultimately, by Nature herself. And at this point, everyone had to agree. In the world of science, it is indeed what happens in the real world that decides which theory is eliminated, while the 'winning' theory is maintained until further notice, until perhaps it needs to be refined or even replaced, due to further discoveries (see later). Note also that, despite the disagreements that may occur along the way, it is indeed *a collective, self-correcting undertaking* which worked perfectly in this case.

Well, that may seem like a bit of a digression, but it's not actually so far away from our main theme. We can see right away why its relevant. To do so, let's ask exactly the same question about matter: is it wavelike or particle-like? This is indeed the question raised by quantum theory. So let's take a look at exactly the same experiment as the one done by Young, but replacing the beam of light by a beam of electrons (see Fig. 17). Today we can indeed produce such a beam, in which all the electrons have the same speed and the same direction.

Normally, if the electrons were particles, we would expect those that were not blocked by the first screen simply to go through one of the two slits, each then producing a detection point on the screen behind it (on the right in the figure), i.e., going straight to the detection screen to produce one of the tiny white points, since that is how each detection is revealed. As these detection points accumulate, one would thus expect to see two parallel white rectangles forming in the image of the two slits. But what we actually see is nothing like that. Take a look at Fig. 18, which shows the result of an experiment by the Japanese physicist Akira Tonomura in 1989. This must be one of the most remarkable photos of the twentieth century.

It is important to note that the electrons can be sent through one by one, such is our level of technological mastery these days. We can see that the electrons are indeed detected as tiny white points on the screen, and we can even watch them arrive and accumulate in the four images of the figure, taken

[4]Future Prime Minister of France! In those days, science had its place in government.

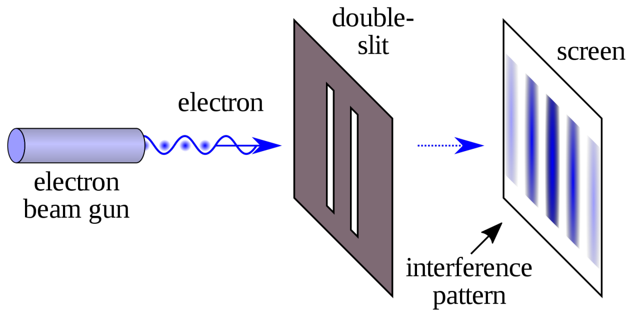

Figure 17: Two-slit experiment, but replacing the light beam by a beam of electrons. We can already see what that's going to give on the detection screen on the right. Credit: en.wikipedia.org/wiki/Double-slit_experiment

after four different time intervals, starting top left. But when we look at the fourth image, bottom right, what we see is a wave interference pattern! It looks like the electrons behaved like a wave to get through the two slits. And yet they look for all the world like particles at the instant when they are detected, at the instant when we choose to *look at* them. This is the famous *wave–particle duality*. Some like to say that the electron is a wave when it goes through the slits and it becomes a particle only when we choose to *look* at it. And recall that there is no influence of one electron on another, since we can arrange for the electrons to go through one by one.

In the last paragraph, I used the verb 'look at' deliberately, scare quotes and all, to draw attention to one of the problems raised by the interpretation of this experiment. Clearly, this way of speaking leaves only a small step to claiming that the mind can act directly on matter at the microscopic level! The answer is that it can't, or at least that there is absolutely no need for such an extreme explanation. In a few moments we will examine a perfectly clear way to understand what is happening to these electrons that involves no mystery at all. It's true that, if we set up some device behind one of the slits to 'see' whether the electron went that way, the fringes immediately disappear. But then that wouldn't be the same experiment, because in that case we intervene *physically* by setting up this device. And once again there is a clear and simple explanation, as we shall see shortly.

But first of all, the time has come to ask: what is a quantum wave? In practice, it is a wave of probability: the probability of *finding* the electron at any given position and any given instant of time. Note already that the use of the word 'finding' here once again requires the intervention of an observer. We shall soon return to this point. But anyway, in practice, the quantum theory tells us the possible results of any given measurement, e.g., the possible energy levels for an electron in a hydrogen atom, but in the usual way of speaking about things, it only gives us the probability of *finding* a given result when we

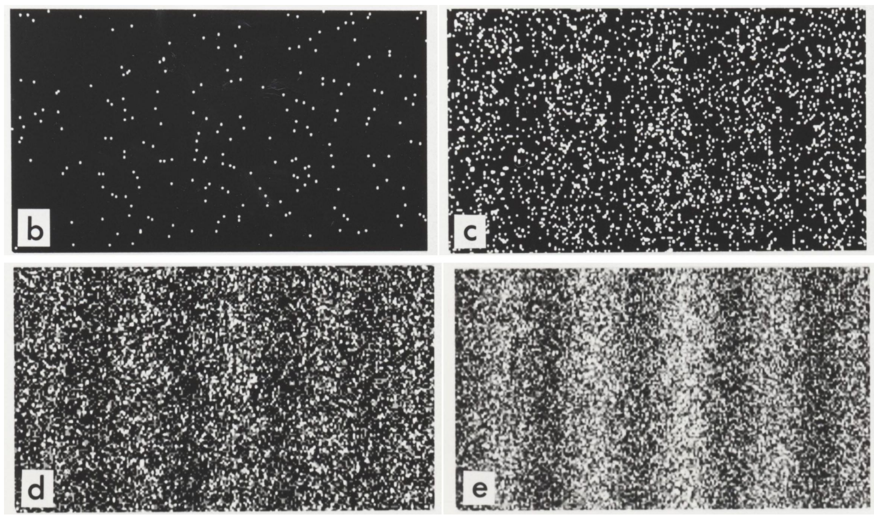

Figure 18: The experiment by Akira Tonomura in 1989. A beam of electrons arrives on a screen containing two slits. This is what appears on the detection screen after four different time intervals, starting top left and ending bottom right. Image public domain

carry out a measurement.

So here seems to be a characteristic feature of the quantum world: it is a probabilistic one, at least regarding the results of measurements made by an observer. But does that mean that the world is therefore necessarily intrinsically indeterminist? Does it mean that, on the microscopic level, effects are not entirely determined by causes? In a moment we shall see that this is not necessarily the case.

And here's another problem. The wave is represented in time and space by a function

$$\psi(\vec{r}, t) \ .$$

This is the *wave function*. At each point in space, indicated by a vector $\vec{r}$ with three components, and at each instant of time t, this function is used to calculate the probability of finding the electron at this point and at this instant of time. For example, look at the fourth image in Fig. 18. There are places where we see a lot of detection points and other places where we see a lot less. These are respectively points where, at this time t, the value given for the probability by ψ is large and small, respectively.

In the context of the two-slit experiment, and in the usual way of teaching quantum theory, it's as though the electron were somehow spread out over a whole region of space until the moment when we choose to actually see where it is, and at this instant, the wave collapses to the point where we have just observed it, i.e., where we are sure to find it again if we take another look right away. This has been referred to as the *collapse of the wave function*.

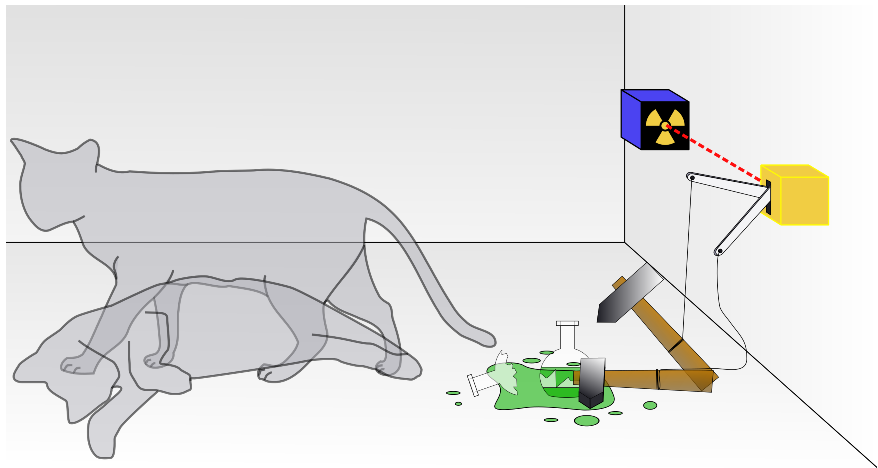

Figure 19: What really happened to Schrödinger's cat? Image public domain

But there's something very strange about this. Normally, when there are no observers around to disturb the electron by trying to detect it, its wave function evolves according to the *Schrödinger equation*:

$$\mathrm{i}\hbar \frac{\partial \psi}{\partial t} = \left(-\frac{\hbar^2}{2m} \nabla^2 + V \right) \psi \ .$$

This is a *wave equation* which governs the way the quantum wave evolves in time. In other words, the Schrödinger equation tells us how the wave behaves up to the time when we make a measurement, but it suddenly evolves in quite a different way when we make the measurement. It instantaneously becomes a wave function that is completely concentrated at the point where we have found the electron, and zero elsewhere. So why is there this difference of behaviour, with such radically different dynamics, when we make a measurement? This is referred to as the *measurement problem*. Here, too, we shall soon examine an explanation with no mystery attached.

But first a word about the cat (see Fig. 19). It was Erwin Schrödinger who had this idea. I should say, don't worry, it's just a thought experiment. Anyway, it would be difficult to actually set it up in practice. And that would be quite unnecessary, because it's just the idea behind it that matters. In fact, this image comes from Wikipedia,[5] but the explanation given there is not the same as the one I'm going to advocate here, and indeed it illustrates the confusion surrounding this famous thought experiment. The following is translated from the French Wiki page:

> A cat is shut in a box with a device that will kill the animal as soon as it detects the decay of a radioactive atom. The device may be a

[5]fr.wikipedia.org/wiki/Chat_de_Schrödinger

> detector like a Geiger counter, for example, connected to a switch. If the counter detects a decay, the switch causes a small hammer to fall and break open a phial containing poison—Schrödinger suggested hydrocyanic acid, which can be stored under pressure in the phial in liquid form and which will instantaneously evaporate, releasing a deadly gas, if ever the phial is broken.
>
> Suppose for the sake of argument that the quantum theory indicates that a decay will occur with a probability of one half by the end of the first minute. Then the theory also tells us that, as long as no observation is made (or more precisely, as long as the wave function has not collapsed), the atom will be simultaneously in two states: intact and decayed. But the mechanism imagined by Erwin Schrödinger links the state of the cat (dead or alive) to the state of the radioactive particle, in such a way that the cat will also be simultaneously in two states (the dead state and the living state), until opening the box (the observation) triggers the choice between the two states. This implies that it is impossible to say whether the cat is dead or alive after one minute.

Clearly, this is nonsense. And it is not the real world, not even the real world according to quantum theory, that is so absurd. It is just this way of putting things, which is unfortunately absolutely typical. The first point is that the atom is not simultaneously in two states after one minute, as claimed above. It is in a *superposition* of states. This is not at all the same thing. But what is a superposition?

If two waves satisfy the equation mentioned a moment ago, the Schrödinger equation, then their sum will also satisfy it. This sum is referred to as the *superposition* of the two waves. I already discussed this in the context of the ripples on water. When we drop two small stones into a pond, the ripples they produce are superposed whenever they meet. Wave equations are often like this: superpositions of solutions are also solutions. Such equations are said to be linear. We have already seen an example of superposition, because the electron in the two-slit experiment is itself in a superposed state! According to quantum theory, the two quantum waves generated at the two slits superpose, just like the ripples on water. And it is often said that the electron is itself 'in' this superposed state, while remaining as vague as possible about what we conceive to be the electron. Well, it's not obvious, is it? What does it mean to say that an electron is 'in' a superposed state?

The above radioactive atom is itself 'in' a superposition. And quite frankly, there's little hope of imagining what that could mean, especially since, in the standard presentation of all this, the *ontology* of the theory, i.e., the list of all things considered to really exist, contains only the wave and no particles! But of course, an electron or an atom is a very small thing. Much too small to be seen with the naked eye. What's more, they belong to the microscopic world, and why should it be easy to understand that world? But the problem with this argument is that, with Schrödinger's cat experiment, we see that, according to the quantum theory, even a cat can be ‘in’ a superposed state, and not just any superposed state! A superposition of a state in which it is alive and a state in which it is dead. And it was precisely this point that Schrödinger wanted to make when he proposed this thought experiment: if microscopic entities can be in these superposed states, then so can macroscopic ones.

We cannot even say, as it is said in the above Wikipedia extract, that the cat is simultaneously in both states, the living state and the dead state. In truth, we just don’t know what to say in this way of looking at things. What does it mean to be in a superposition when one is a cat? We don’t know. (And the same can be said for an atom if we’re honest!) But as you can see in the continuation of the above extract, the situation is worse than that, because its author concludes that opening the box triggers the choice between the two states. In other words, it’s the observation that triggers it. The observer’s choice to intervene. Until then, the cat remains in a state that’s impossible to imagine, but only because no one is looking! Mind over matter again, and even mind deciding matters of life and death!

So, what about a solution to all these puzzles? Let’s go back to the hero of this story, Louis Victor de Broglie, prince, then Duke de Broglie! Because he was the one who suggested, way back when quantum theory was still in its infancy in the 1920s, that matter might have certain wavelike features. However, he didn’t say that the electron *was* a wave. He said that the electron was perhaps being *guided* by a wave, which is not at all the same thing. Unfortunately, this idea was abandoned under the pressure of certain well established scientists like Bohr. That’s a whole story in itself! And then around 1950, an American physicist, David Bohm, took up the idea again and eventually inspired others to do the same, including John Bell, Sheldon Goldstein, Detlef Dürr, Nino Zanghì, and many others who have since developed this theory, which is often called *Bohmian mechanics*.

We should say right away that the electron in the two-slit experiment is guided by a wave which is a superposition of the two circular waves generated at the two slits. It is indeed for this reason that there is a greater probability of arriving at the detection screen at a point where these two waves superpose positively, at the superposition of two peaks, and a lower probability of arriving at the detection screen at a point where these two waves cancel one another, at the superposition of a peak and a valley.[6] Supplementing the Schrödinger equation by another entirely deterministic equation, this theory predicts exactly where each electron will end up, given its initial position in the beam. Figure 20 shows some of the solutions of this supplementary equation, called the *guiding equation*. In other words, it shows some of the possible trajectories of the electron. It is thus an entirely deterministic theory, just like Newton’s theory of motion! The predictions of the theory are probabilistic because, before it enters the two-slit setup, there is a whole range of possible positions for the electron across the whole width of the electron beam and each will yield a different

[6] As a technical point, it is the square of the wave function that gives the probability, so the probability is also high at points where two valleys are superposed.

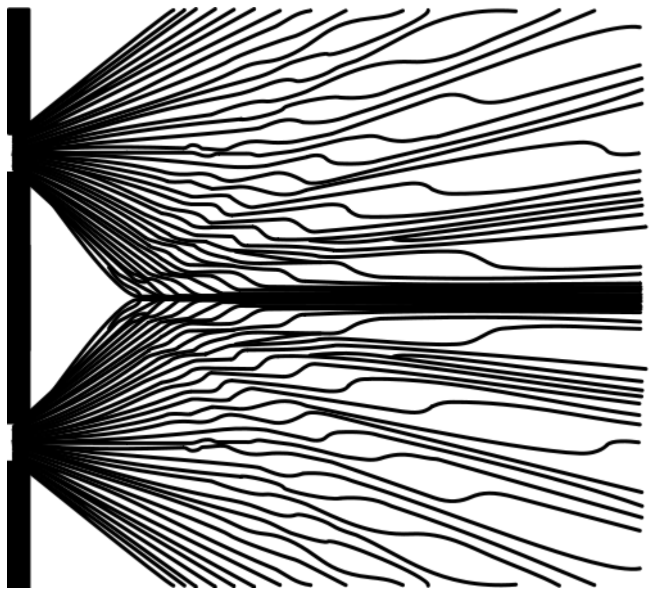

Figure 20: Possible trajectories of the electron in the two-slit experiment, according to Bohmian mechanics. The exact trajectory of an electron depends on its initial position in the beam. Image public domain

trajectory.

There is no difficulty no in explaining why what is detected on the screen behind the two slits is always a tiny point. It's because we detect a tiny particle! In the ontology of this theory, the list of things that really exist according to this theory, there are particles! We don't even need to put the detection screen there to know that the electrons will be somewhere, and if we do put it, we don't need to actually look at the detection point for the electron to be there where there is a tiny white point! No observer is necessary in this theory. And the probability given by the wave function is not just the probability of *finding* the electron at this or that point, but the probability that the electron *is* at each of these points. Moreover, when we try to 'see' if the electron went through one particular slit, the interference fringes disappear because the wave that guides the electron is no longer a superposition of two circular waves, and this because we changed the experimental setup. No mysteries.

There is no measurement problem in the Bohmian theory. Recall that we were bothered by the fact that the wave function seems to collapse suddenly when we look at the detection screen, whereas at any other moment, the wave function evolves meekly according to Schrödinger's equation. In the Bohmian theory, there is never a collapse. The function always evolves according to Schrödinger's equation. The explanation for the apparent disappearance of a part of the wave function once the detection screen has done its work (and this without any observer having to look at the result in this case) is simply that a good part of the wave function can never again influence what subsequently happens to the electron, i.e., all those parts of the wave function that would have corresponded to detection elsewhere on the detection screen. In Bohmian mechanics, one speaks of the *effective* wave function, but of course, the details are a little technical.[7] Another bonus with this theory is that it shows us exactly what happens to particles whenever we measure a physical quantity associated with them, as one sees explicitly in the simulation of Fig. 20, but also in every other case.

So how does this help us to understand what happens to the cat? According to the theory, after one minute, the radioactive atom is in its superposed state, i.e., it is now *guided* by a wave function that is the sum of a wave function for an intact atom and a wave function for a decayed atom. The atom itself will of course be either intact or decayed, and the cat will be either alive or dead, respectively. There's no problem understanding what it means for an entity, even a macroscopic one, to be in a superposition state: the theory tells us how the superposition must *guide* the given entity. Moreover, no need to look in the box for things to be as clear as this. What simplifies and clarifies all this is the fact that . . . the cat is there! It is made of particles and this theory has particles in its ontology. When we only allow ourselves the waves, we soon get into deep trouble! The above Wikipedia extract proves the point.

So far I've explained why we invented the quantum theory: to explain observations of the way light is emitted and absorbed by matter. We have seen a model for matter: it was the return of the atom. So the quantum theory is above all a theory of the atom and the interaction between atoms and light. It's important to note that this is a very precise and practical theory which explains the periodic table of the elements (see Fig. 21), i.e., it explains in principle the properties of all the chemical elements, and today we know all the chemical elements that can exist naturally. So this theory explains in principle all the interactions between matter and matter, i.e., it explains chemistry. It makes very precise predictions, although some of its explanations are perhaps rather novel! I went on to sketch what is strange in quantum physics, or at least, what is really *new* in this theory. Microscopic particles also have a wavelike behaviour, as we have seen with the two-slit experiment. I then described the measurement problem and the solution provided by Bohm and de Broglie.

So now, let us talk about what is *really* strange in this theory! In fact, in both these theories, quantum mechanics and Bohmian mechanics. The two theories agree on all predictions, so I shall just say 'the' theory in the following. (They differ significantly in their explanatory power, as we have seen.) This is the third part of the presentation and the subject is *quantum nonlocality*. In the 1920s, the formalism of quantum theory was developed and extended. But, by 1935, a rather awkward problem had already come to light. The theory looked as though it made a pretty strange prediction, one that might even contradict the theory of relativity, one of the pillars of fundamental physics.

[7]The first paper explaining this in detail was: Dürr, D., Goldstein, S., and Zanghi, N. (1992), Quantum equilibrium and the origin of absolute uncertainty, Journal of Statistical Physics **67**, 843–907. It is freely available at `arxiv.org/abs/quant-ph/0308039`

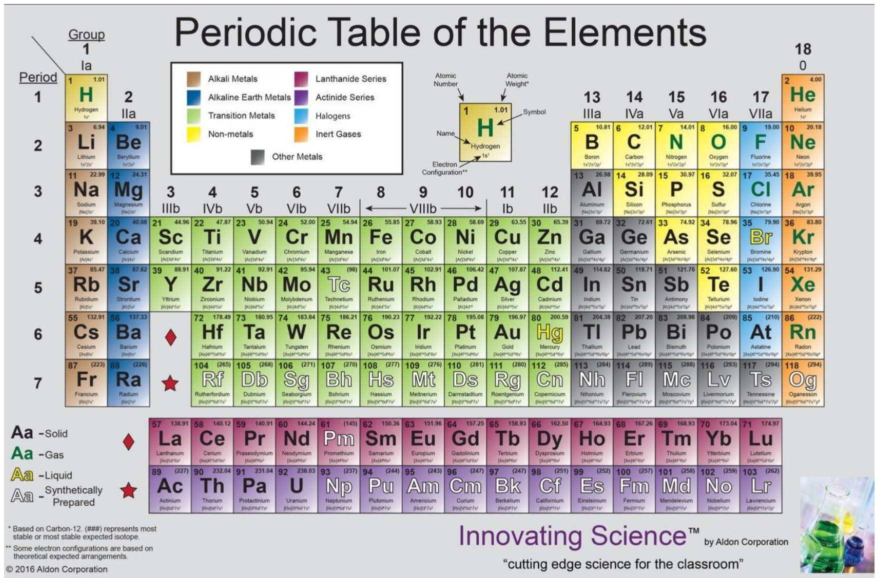

Figure 21: The periodic table of the elements. The table shows all the elements that can exist, and some others that cannot exist naturally. Image from the website `www.arborsci.com/products/periodic-table-of-the-elements-poster-size`

The theory predicts that, in certain situations involving microscopic particles, if I do something here, I instantaneously influence something else far from here. The first experimental tests were devised in the 1970s. It took a long time to get that far because such tests are no easy matter to set up:

- Experiments in Harvard and Berkeley in the United States from 1972
- Alain Aspect in France in the period 1975–1980 (signal $> 2c$)
- Nicolas Gisin in Geneva in 1998 (signal $> 10^7 c$)

I'll explain the notes about the signal speed in brackets in a moment (c is the speed of light). I'd like to illustrate the phenomenon by giving a very superficial description of Gisin's experiment, since it is one of the most striking. A warning though: we're going to talk about photons, particles of light! Because, you've guessed it, light sometimes behaves in a corpuscular way, just as Newton and Descartes had always said (but they were a long way from understanding what we know about this today). And remember, I did say earlier that no scientific hypothesis is ever completely safe from refinement or replacement.

So what about Gisin's experiment? The idea was to shine infrared light on a special kind of crystal. Spontaneously, from time to time, the crystal would emit a pair of green photons which would fly out back to back, i.e., in opposite directions. Now it turns out that such photons are in an *entangled* quantum state. Each photon was picked up in an optical fibre and carried several kilometers from its origin and from the other photon, to two villages outside Geneva. Call the photons A and B. We now measure the polarisation of one of the two photons, let's say A. It doesn't really matter what the polarisation is. Let's just say that it's a physical quantity associated with each photon and which has an orientation in space.

When we measure the polarisation of photon A in a certain direction, the value will always be either 1 or 0, i.e., we will always find it to be either in the chosen direction of measurement or else perpendicular to it. Note that this is a typically quantum kind of result: there are only two possible values. When we measure a classical (non-quantum) physical quantity, such as the orientation of a macroscopic object in a given direction, we can obtain a whole range of possible values. Note also that only Bohmian mechanics can explain why there will be these two possible values, whatever direction we choose to measure in. So we now repeat this experiment thousands of time, and each time we measure the polarisation of photon A in a certain direction, it doesn't matter which, but for the moment always in the same direction. We don't know beforehand what value will be found for the polarisation, but for whatever direction we have chosen, after many measurements, we will find that half the photons A will give a value in this direction and the other half will give a value in the perpendicular direction. The result is as random as random can be.

Now, whenever two photons come out of the crystal back to back in this entangled quantum state, instead of measuring the polarisation of just one of them, we measure the polarisations of both. Each time, we choose a direction for A at random and another direction for B at random. We choose these two directions at the last moment, just as they arrive in their respective villages, and simultaneously, and each time, for each photon, whatever directions have been chosen, two values are possible: 0 or 1. The idea is that there should be no way for a signal to pass between the two photons during the measurements, because such a signal would have to travel faster than light. But the thing is, we find a correlation between the results of these measurements, even when they are made miles apart. And what's more, it's exactly the correlation predicted by quantum theory!

So there is some kind of nonlocal connection, a direct influence between two simultaneous events in two different places. This is a prediction that distinguishes quantum theory from all our classical theories . . . and indeed from all our intuitions. What's more, this influence does not weaken with the distance between the two particles. And this too distinguishes quantum theory from our classical theories, and even more from our intuitions. These experiments are difficult because entangled states are rather delicate. Moreover, one must eliminate all possibility of common causes that might produce the observed correlations. That's why, in these experiments, the detectors measuring the polarisations of A and B are oriented:

- at the last moment,
- at random,
- simultaneously.

But how to make two measurements in different places at exactly the same time? Of course, two such measurements in two different places will never be *exactly* simultaneous. Aspect's experiment was accurate enough to show that, if there were some kind of signal travelling between the measurements of A and B, it would have to go at least twice as fast as light (see the comments in brackets above). As the two photons were measured at a separation of a several kilometers in Gisin's experiment, he was able to show that a signal travelling between the measurements of A and B would have to travel at least ten million times faster than light (10^7c, where c is the speed of light).

I said above that there had to be a direct *influence*, and not a signal, because we imagine a signal as something that goes from point to point through space along some trajectory, which would be impossible if the signal had to arrive instantaneously, at exactly the same time as it left. For one thing, the theory of relativity forbids any signal from going faster than light. This is why, in the experiments of Aspect and Gisin, the approximation to simultaneity in the measurements on A and B is already good enough at least to make relativists (advocates of the theory of relativity) feel uneasy! But fortunately for the latter, it turns out that one cannot use these influences to send messages faster than light. This is basically for the simple reason that the results of polarisation measurements are perfectly unpredictable. One says that these experiments do not outright contradict relativity theory, even though they do indeed go against the spirit of that theory.

There is one other rather fascinating thing regarding this conflict with the theory of relativity. According to the latter, the notion of simultaneity is itself relative. What this means is that, if I carry out two measurements M_1 and M_2 which I consider to be simultaneous, in two different places, another observer[8] in uniform motion relative to me will consider that one of those measurements was made before the other, let's say M_1 before M_2. And a third observer in uniform motion relative to me but in the opposite direction to the second observer, will consider that the measurement M_2 was made before M_1. So this influence I've been speaking about, what direction does it go? From M_1 to M_2 or from M_2 to M_1? The answer is that it doesn't *go* at all, it just *is*.

> Before I came here I was confused about this subject. Having listened to your lecture I am still confused. But on a higher level.
>
> Enrico Fermi (1901–1954)

[8]In Bohmian mechanics, we get rid of the omnipresent observer of quantum mechanics, but we keep it in relativity theory!